Inter (Part-I) 2017

Mathematics		PAPER: I
Time: 30 Minutes	(OBJECTIVE TYPE)	Marks: 20

Note: Four possible answers, A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1- **^nC_r is equal to:**

(a) $\frac{n!}{r!}$ (b) $\frac{n!}{(n-r)!}$

(c) $n(n-r)!$ (d) $\frac{n!}{r!(n-r)!}$ ✓

2- **$\frac{1}{4}$ rotation (anti-clockwise) = :**

(a) 45° (b) 90° ✓

(c) 180° (d) 360°

3- **Notation for radius of in-circle is:**

(a) r ✓ (b) R

(c) r_1 (d) Δ

4- **The value of cos 315° is:**

(a) 0 (b) 1

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$ ✓

5- **Harmonic mean between 3 and 7 is:**

(a) $\frac{5}{21}$ (b) $\frac{21}{5}$ ✓

(c) 5 (d) 21

6- **Period of $\tan\frac{x}{2}$ is:**

(a) π (b) 2π ✓

(c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

7- A quadratic equation has degree:
(a) 0 (b) 1
(c) 2 √ (d) 3

8- Set of integers is a group with respect to:
(a) + √ (b) ÷
(c) × (d) –

9- Number of terms in the expansion of $(1+x)^{2n+1}$ is:
(a) $2n+1$ (b) $2n$
(c) $2n+2$ √ (d) $3n+1$

10- The sum of odd coefficient in the expansion of $(1+x)^5$ is :
(a) 5 (b) 16 √
(c) 25 (d) 32

11- Arithmetic mean between $\frac{1}{a}$ and $\frac{1}{b}$ is:
(a) $\frac{a+b}{2ab}$ √ (b) $\frac{a+b}{ab}$
(c) $\frac{2ab}{a+b}$ (d) $\frac{ab}{a+b}$

12- If A is a matrix of order 3×4, then order of AA^t is:
(a) 4×3 (b) 3×4
(c) 4×4 (d) 3×3 √

13- Partial fractions of $\frac{1}{x^2-1}$ will be of the form:
(a) $\frac{Ax+B}{x^2-1}$ (b) $\frac{A}{x+1}+\frac{B}{x-1}$ √
(c) $\frac{A}{x+1}$ (d) $\frac{B}{x-1}$

14- The roots of equation $x^2-5x+6=0$ are:
(a) $2, -3$ (b) $-2, -3$
(c) $2, 3$ √ (d) $-2, 3$

15- $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) =$:
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ √
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

16- $\sqrt{\frac{s(s-a)}{bc}}$ equals:

(a) $\sin\frac{\alpha}{2}$
(b) $\sin\frac{\beta}{2}$
(c) $\cos\frac{\alpha}{2}$ ✓
(d) $\cos\frac{\beta}{2}$

17- $\cos x = \frac{1}{2}$ has solution ----- $x \in [0, \pi]$:

(a) $\frac{\pi}{6}$
(b) $\frac{\pi}{3}$ ✓
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{2}$

18- If $\begin{vmatrix} k & 4 \\ 4 & k \end{vmatrix} = 0$, then k = ----- :

(a) 16
(b) 0
(c) ± 4 ✓
(d) 8

19- $\frac{3!}{0!}$ equals:

(a) 3
(b) 6 ✓
(c) ∞
(d) 12

20- If $z = 3 - 4i$, then $|\bar{z}|$ is:

(a) 4
(b) 5 ✓
(c) -5
(d) 1

Inter (Part-I) 2017

Mathematics		PAPER: I
Time: 2.30 Hours	(SUBJECTIVE TYPE)	Marks: 80

SECTION-I

2. Write short answers to any EIGHT (8) questions: 16

(i) Does the set $\{0, -1\}$ possess closure property with respect to:
(a) addition (b) multiplication

Ans (a)

Since $(-1) + (-1) = -2 \notin \{0, -1\}$,
so $\{0, -1\}$ is not closed w.r.t addition.

(b)

Since $(-1) \times (-1) = 1 \notin \{0, -1\}$,
So $\{0, -1\}$ is not closed w.r.t multiplication.

(ii) Find multiplication inverse of a + bi.

Ans For multiplicative inverse, the reciprocal of a and b is:

$$\frac{1}{(a)^2 + (b)^2} + \frac{-b}{(a)^2 + (b)^2} i = \frac{1}{a^2 + b^2} - \frac{b}{a^2 + b^2} i$$

(iii) Prove that $|z_1 z_2| = |z_1| \, |z_2| \ \forall \ z_1, z_2 \in C$

Ans $\text{L.H.S} = |z_1 \, z_2|$

As we known that:

$$z_1 = a + ib, \ z_2 = c + id, \text{ then}$$

$$|z_1, z_2| = |(a + ib)(c + id)|$$
$$= |(ac - bd) + (ad + bc)\, i|$$
$$= \sqrt{(ac - bd)^2 + (ad + bc)^2}$$
$$= \sqrt{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}$$
$$= \sqrt{(a^2 + b^2)(c^2 + d^2)}$$
$$= |z_1| \cdot |z_2|$$

This result may be stated thus:

The modulus of the product of two complex numbers is equal to the product of their moduli.

(iv) Define proper subset and improper subset.

Ans **Proper Subset:**

If A is a subset of B and B contains at least one element which is not an element of A, then A is said to be a proper subset of B. In such a case, we write:

$A \subset B$ (A is a proper subset of B)

Improper Subset:

If A is subset of B and A = B, then we say that A is an improper subset of B. From this definition, it also follows that every set A is an improper subset of itself.

(v) Show that the statement is tautology $\sim(p \to q) \to p$.

Ans

p	q	$p \to q$	$\sim(p \to q)$	$\sim(p \to q) \to p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

Since all the possible values of $\sim(p \to q) \to p$ are true. Thus $\sim(p \to q) \to p$ is a tautology.

(vi) If (G, $\cdot\dot{X}\cdot$) is a group with 'e' its identity then 'e' is unique?

Ans Suppose the contrary that identity is not unique. And let e' be another identity.

e, e' being identities, we have

$e' \cdot\dot{X}\cdot e = e \cdot\dot{X}\cdot e' = e'$ (e is an identity) (i)

$e' \cdot\dot{X}\cdot e = e \cdot\dot{X}\cdot e' = e$ (e' is an identity) (ii)

By comparing (i) and (ii), we get

$e' = e$

Thus the identity of a group is always unique.

(vii) $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, show that $A^4 = I_2$.

Ans

$$A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$A^2 = A.A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$= \begin{bmatrix} i(i) + 0(1) & i(0) + 0(-i) \\ 1(i) + (-i)(1) & 1(0) + (-i)(-i) \end{bmatrix}$$

$$= \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^4 = A^2 . A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1(-1) + 0(0) & -1(0) + 0(-1) \\ 0(-1) + (-1)(0) & 0(0) + (-1)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I_2$$

$A^4 = I_2$ Proved

(viii) $A = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix}$, show that $A - (\bar{A})^t$ is skew-hermitian.

Ans Given, $A = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix}$

$$\bar{A} = \begin{bmatrix} -i & 1-i \\ 1 & i \end{bmatrix}$$

$$(\bar{A})^t = \begin{bmatrix} -i & 1 \\ 1-i & i \end{bmatrix}$$

Let,

$$Y = A - (\bar{A})^t$$

$$= \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix} - \begin{bmatrix} -i & 1 \\ 1-i & i \end{bmatrix} = \begin{bmatrix} 2i & i \\ i & -2i \end{bmatrix}$$

Now,

$$(\bar{Y})^t = \begin{bmatrix} -2i & -i \\ -i & 2i \end{bmatrix}^t$$

$$= \begin{bmatrix} -2i & -i \\ -i & 2i \end{bmatrix}$$

$$= -Y$$

Thus $Y = A - (\bar{A})^t$ is skew-hermitian.

(ix) Without expansion show that $\begin{vmatrix} bc & ca & ab \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ a & b & c \end{vmatrix} = 0.$

Ans L.H.S = $\begin{vmatrix} bc & ca & ab \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ a & b & c \end{vmatrix}$

Multiplying all elements of second row by 'abc', we have

$$= \frac{1}{abc}\begin{vmatrix} bc & ca & ab \\ \frac{abc}{a} & \frac{abc}{b} & \frac{abc}{c} \\ a & b & c \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} bc & ca & ab \\ bc & ca & ab \\ a & b & c \end{vmatrix}$$

Since all elements of 1st row and 2nd row are identical, so

$$= \frac{1}{abc}(0)$$
$$= 0$$
$$= \text{R.H.S}$$

(x) Solve the equation $x^{1/2} - x^{1/4} - 6 = 0.$

Ans This given equation can be written as:

$$(x^{1/4})^2 - x^{1/4} - 6 = 0$$

Let $x^{1/4} = y$

∴ The given equation becomes

$$y^2 - y - 6 = 0$$
$$(y - 3)(y + 2) = 0$$
$$y - 3 = 0 \quad ; \quad y + 2 = 0$$
$$y = 3 \quad ; \quad y = -2$$

As $x^{1/4} = y$

So, $x^{1/4} = 3$

$$(x^{1/4})^4 = (3)^4$$
$$x = 81$$

and $x^{1/4} = y$

$$x^{1/4} = -2$$
$$(x^{1/4})^4 = (-2)^4$$
$$x = 16$$

Hence the solution set is $\{16, 81\}$

(xi) **When $x^3 + kx^2 - 7x + 6$ is divided by $x + 2$ the remainder is -4? Find the value of k.**

Ans Let $f(x) = x^3 + kx^2 - 7x + 6$

and $x - a = x + 2$, we have

$a = -2$

(By Remainder Theorem)

$$\begin{aligned}\text{Remainder} &= f(-2)\\ &= (-2)^3 + k(-2)^2 - 7(-2) + 6\\ &= -8 + 4k + 14 + 6\\ &= 4k + 12\end{aligned}$$

Given that remainder $= -4$

$$\begin{aligned}\therefore \quad 4k + 12 &= -4\\ 4k &= -4 - 12\\ 4k &= -16\\ k &= -4\end{aligned}$$

(xii) **Prove that $1 + \omega + \omega^2 = 0$.**

Ans We know that cube roots of unity are:

$$1, \frac{-1 + \sqrt{3}\,i}{2} \text{ and } \frac{-1 - \sqrt{3}\,i}{2}$$

If $\omega = \frac{-1 + \sqrt{3}\,i}{2}$,

then $\omega^2 = \frac{-1 - \sqrt{3}\,i}{2}$

Sum of all the three cube roots

$$\begin{aligned}1 + \omega + \omega^2 &= 1 + \frac{-1 + \sqrt{3}\,i}{2} + \frac{-1 - \sqrt{3}\,i}{2}\\ &= \frac{2 - 1 + \sqrt{3}\,i - 1 - \sqrt{3}\,i}{2}\\ &= \frac{0}{2} = 0\end{aligned}$$

Hence sum of cube roots of unity

$1 + \omega + \omega^2 = 0$

3. Write short answers to any EIGHT (8) questions: 16

(i) **Resolve $\frac{1}{x^2 - 1}$ into partial fractions.**

Ans

$$\frac{1}{x^2 - 1} = \frac{1}{(x + 1)(x - 1)}$$

$$\frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$1 = A(x-1) + B(x+1) \qquad (1)$$

Put $x + 1 = 0$

$$1 = -2A$$

$$A = -\frac{1}{2}$$

Now put $x - 1 = 0$

$x = 1$ in (1), we get.

$$1 = 2B$$

$$B = \frac{1}{2}$$

Now,

$$\frac{1}{(x+1)(x-1)} = \frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}$$

$$= -\frac{1}{2(x+1)} + \frac{1}{2(x-1)}$$

Which are required partial fractions.

(ii) **If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in G.P, show that $r = \pm\sqrt{\frac{a}{c}}$.**

Ans Given $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in G.P

Let r be the common ratio of the G.P

$$\therefore \quad r = \frac{\frac{1}{b}}{\frac{1}{a}} = \frac{a}{b} \qquad (i)$$

Also $$r = \frac{\frac{1}{c}}{\frac{1}{b}} = \frac{b}{c} \qquad (ii)$$

Multiply (i) and (ii),

$$r^2 = \frac{a}{b} \times \frac{b}{c}$$

$$r = \pm\sqrt{\frac{a}{c}}$$

(iii) **Convert recurring decimal 0.7 into vulgar fraction.**

Ans $0.7 = 0.7777$ ....

$$= 0.7 + 0.07 + 0.007 + 0.0007 + \dots\dots$$

$$= \frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \frac{7}{10,000} + \dots\dots$$

Here, $a = \frac{7}{10}$,

$$r = \frac{7}{100} \times \frac{10}{7}$$

$$= \frac{1}{10}$$

$$\therefore \quad S_\infty = \frac{\left(\frac{7}{10}\right)}{1 - \frac{1}{10}} = \frac{7}{9}$$

(iv) If 5 is the harmonic mean between 2 and b, find b?

Ans Here, $a = 2, b = b$

We know that

$$H.M = \frac{2ab}{a + b}$$

By given condition,

$$\Rightarrow \quad H.M = \frac{2(2)(b)}{2 + b} = 5$$

$$\Rightarrow \quad \frac{4b}{2 + b} = 5$$

$$4b = 5(2 + b)$$

$$4b = 10 + 5b$$

$$4b - 5b = 10$$

$$-b = 10$$

$$\boxed{b = -10}$$

(v) Find the A.P. whose nth term is 3n – 1.

Ans Given, $a_n = 3n - 1$

Substituting n = 1, 2, 3, 4 and so on.

For $n = 1$	$a_1 = 3(1) - 1 = 2$
$n = 2$	$a_2 = 3(2) - 1 = 5$
$n = 3$	$a_3 = 3(3) - 1 = 8$
$n = 4$	$a_4 = 3(4) - 1 = 11$

and so on.

Therefore, the required A.P is 2, 5, 8, 11, ... , $3n - 1$.

(vi) How many words can be formed from the letters of the word 'Objective' using all letters without repeating any one?

Ans We have to form permutation of 9 letters taken 9 at a time.

$$^9P_9 = 9!$$
$$= 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$
$$= 362,880$$

(vii) In how many ways 4 keys can be arranged on a circular key ring?

Ans 4 keys can be arranged on a circular key ring in $\frac{1}{2}(3!)$ or 3 ways.

(viii) Find the values of n and r when $^nC_r = 35$ and $^nP_r = 210$.

Ans

$$^nC_r = 35$$
$$\frac{n!}{(n-r)!\,r!} = 35 \qquad (1)$$

$$^nP_r = 210$$
$$\frac{n!}{(n-r)!} = 210 \qquad (2)$$

Using eq. (2) in eq. (1),

$$\frac{210}{r!} = 35$$
$$\Rightarrow \quad r! = \frac{210}{35}$$
$$r! = 6$$
$$r! = 3!$$
$$\boxed{r = 3}$$

Put in (2),

$$\frac{n!}{(n-3)!} = 210$$
$$\frac{n!}{(n-3)!} = \frac{2 \times 3 \times 7 \times 5 \times 1 \times 4 \times 6}{1 \times 4 \times 6}$$
$$\frac{n!}{(n-3)!} = \frac{7!}{4!}$$
$$\frac{n!}{(n-3)!} = \frac{7!}{(7-3)!}$$

$$^nP_3 = {^7P_3}$$

$$\boxed{n = 7}$$

(ix) If $S = \{1, 2, 3, ----, 9\}$, Events $A = \{2, 4, 6, 8\}$, $B = \{1, 3, 5\}$, find $P(A \cup B)$.

Ans

$$S = \{1, 2, 3, ----, 9\}$$
$$n(S) = 9$$
$$A = \{2, 4, 6, 8\}$$
$$B = \{1, 3, 5\}$$
$$A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$$
$$n(A \cup B) = 7$$
$$\therefore \quad P(A \cup B) = \frac{n(A \cup B)}{n(S)}$$
$$= \frac{7}{9}$$

(x) Prove that $1 + 5 + 9 + ---- + (4n - 3) = n(2n - 1)$, for $n = 1$, and $n = 2$.

Ans For $n = 1$,

$$L.H.S = R.H.S = 1$$

For $n = 2$,

$$L.H.S = R.H.S = \frac{3}{2}$$

(xi) Expand up to three terms $(1 - x)^{1/2}$.

Ans

$$(1 - x)^{1/2} = 1 + \left(\frac{1}{2}\right)(-x) + \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)}{2!}(-x)^2 + \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - 2\right)}{3!}(-x)^3 + ----$$

$$= 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 ------- \text{valid if } |x| < 1.$$

(xii) Using binomial theorem, calculate $(0.97)^3$.

Ans

$$(0.97)^3 = (1 - 0.03)^3$$

$$= \binom{3}{0}(1)^3(-0.03)^0 + \binom{3}{1}(1)^2(-0.03)^1 + \binom{3}{2}(1)^1(-0.03)^2 + \binom{3}{3}(1)^0(-0.03)^3$$

$$= 1 + 3 \times (-0.03) + 3 \times (0.0009) - 1 \times 0.000027$$

$= 1 - 0.09 + 0.0027 - 0.000027$

$= 0.9127$

4. Write short answers to any NINE (9) questions: 18

(i) Find l, when $\theta = \pi$ radians $r = 6$ cm.

Ans As we know that

$l = r\theta$

By putting the given values, we get

$l = 6\pi$

$l = 18.85$ cm

(ii) Verify $\cos 2\theta = 2\cos^2\theta - 1$, when $\theta = 30^\circ, 45^\circ$

Ans

$\cos 2\theta = 2\cos^2\theta - \sin^2\theta - \cos^2\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

When $\theta = 30^\circ$

$\cos 60^\circ = \cos^2 30^\circ - \sin^2 30^\circ$

$\frac{1}{2} = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$\frac{1}{2} = \frac{3}{4} - \frac{1}{4}$

$= \frac{2}{4}$

$= \frac{1}{2}$

So, it is proved that

$\cos 2\theta = 2\cos^2\theta - 1$ when $\theta = 30^\circ$

Again, $\theta = 45^\circ$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 90^\circ = \cos^2 45^\circ - \sin^2 45^\circ$

$0 = \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$

$0 = \frac{1}{2} - \frac{1}{2}$

$= 0$

Hence it is proved that:

$\cos 2\theta = 2\cos^2\theta - 1$ when $\theta = 45^\circ$

(iii) Prove the identity $\frac{1 - \sin\theta}{\cos\theta} = \frac{\cos\theta}{1 + \sin\theta}$.

Ans L.H.S $= \frac{1 - \sin\theta}{\cos\theta}$

$$= \frac{1 - \sin\theta}{\cos\theta} \times \frac{1 + \sin\theta}{1 + \sin\theta}$$

$$= \frac{1 - \sin^2\theta}{\cos\theta\,(1 + \sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta\,(1 + \sin\theta)}$$

$$= \frac{\cos\theta}{1 + \sin\theta}$$

$= \text{R.H.S}$

(iv) If α, β, γ are angles of triangle ABC, then prove that $\tan(\alpha + \beta) + \tan\gamma = 0$.

Ans $\tan(\alpha + \beta) + \tan\gamma = 0$ (i)

$\because \quad \alpha + \beta + \gamma = 180^\circ$

$\alpha + \beta = 180^\circ - \gamma$

Put in (i),

$\tan(180^\circ - \gamma) + \tan\gamma = 0$

$\tan(-\gamma) + \tan\gamma = 0$

$-\tan\gamma + \tan\gamma = 0$

$0 = 0$

L.H.S = R.H.S

(v) Prove that $\cos(\alpha + 45^\circ) = \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)$.

Ans

$$\cos(\alpha + 45^\circ) = \cos\alpha\cos 45^\circ - \sin\alpha\sin 45^\circ$$

$$= \cos\alpha \cdot \frac{1}{\sqrt{2}} - \sin\alpha \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)$$

(vi) Express $2\sin 5\theta\cos 2\theta$ as sum or difference.

Ans

$$2\sin 5\theta\cos 2\theta = \sin(5\theta + 2\theta) + \sin(5\theta - 2\theta)$$

$$= \sin 7\theta + \sin 3\theta$$

(vii) Find the period of $\cos\frac{x}{6}$.

Ans

$$\cos\frac{x}{6} = \cos\left(\frac{x}{6} + 2\pi\right)$$

$$= \cos\frac{1}{6}(x + 12\pi)$$

Hence period of $\cos\frac{x}{6}$ is 12π.

(viii) **In a right angle triangle ABC, a = 5429, c = 6294 and $\gamma = 90^\circ$. Find b, α.**

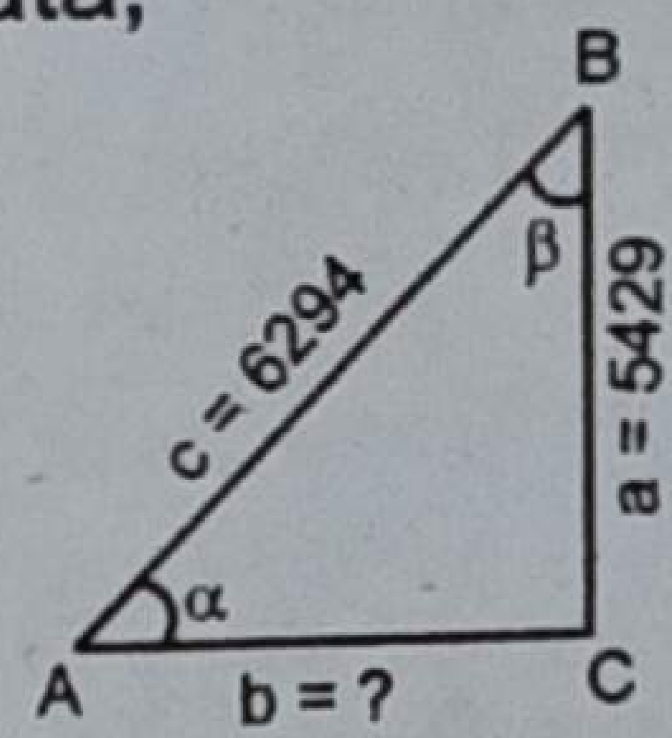
Ans Given,

$$a = 5429$$
$$c = 6294$$
$$\gamma = 90^\circ$$

To find $b = ?$

$$\alpha = ?$$

From the above data,



From figure,

$$\sin\alpha = \frac{5429}{6294}$$
$$\sin\alpha = 0.862567$$
$$\alpha = \sin^{-1}(0.862567)$$
$$\alpha = 59.606^\circ$$
$$\boxed{\alpha = 59^\circ 36'}$$

And by Pythagora's Theorem

$$c^2 = b^2 + a^2$$
$$c^2 - a^2 = b^2$$
$$\Rightarrow \quad b^2 = c^2 - a^2$$
$$b^2 = (6294)^2 - (5429)^2$$
$$b^2 = 10140395$$
$$\boxed{b = 3184.398}$$

(ix) **Define the term circum-circle.**

Ans The circle passing through the three vertices of a triangle is called a circum-circle.

(x) **Find the area of triangle ABC if a = 524, b = 276, c = 315.**

Ans Given, $a = 524, b = 276, c = 315$

$$s = \frac{a+b+c}{2}$$
$$= \frac{524+276+315}{2}$$
$$= \frac{1115}{2}$$
$$s = 557.5$$
$$s-a = 33.5,\ s-b = 281.5,\ s-c = 242.5$$

By area formula,

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{557.5(33.5)(281.5)(242.5)}$$
$$= 35705.894 \text{ square units}$$

(xi) Show that $\cos(\sin^{-1} x) = \sqrt{1-x^2}$.

Ans Given,

$$\cos(\sin^{-1} x) = \sqrt{1-x^2}$$
$$\text{L.H.S} = \cos(\sin^{-1} x)$$

Let $\sin^{-1} x = \theta$

$$x = \frac{1}{\sin}\theta$$
$$x = \cos\theta$$

As $\cos\theta = \sqrt{1-(\sin\theta)^2}$

$$x = \sqrt{1-(\sin\theta)^2}$$

As $\theta = \sin^{-1}(x)$

$$= \sqrt{1-[\sin(\sin^{-1}(x))]^2}$$

As $\theta = \sin[\sin^{-1}(\theta)] = \sqrt{1-x^2}$

$$= \text{R.H.S}$$

(xii) Find solutions of $\operatorname{cosec}\theta = 2, \theta \in [0, 2\pi]$.

Ans

$$\operatorname{cosec}\theta = 2$$

or $\frac{1}{\operatorname{cosec}\theta} = \frac{1}{2}$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\therefore$ $\sin\theta$ is positive in first and second quadrants with the angle $\theta = \frac{\pi}{6}$.

$\therefore \theta = \frac{\pi}{6}$

and $\theta = \pi - \frac{\pi}{6}$

$\theta = \frac{5\pi}{6}$

$\frac{\pi}{6}, \frac{5\pi}{6}$

(xiii) Solve $2\sin\theta + \cos^2\theta - 1 = 0, \theta \in [0, \pi]$.

Ans

$$2\sin\theta + \cos^2\theta - 1 = 0$$
$$2\sin\theta - (1 - \cos^2\theta) = 0$$
$$2\sin\theta - \sin^2\theta = 0$$
$$\sin\theta(2 - \sin\theta) = 0$$

$\therefore \sin\theta = 0$; $2 - \sin\theta = 0$

$\theta = \sin^{-1} 0$; $2 = \sin\theta$

$\theta = 0, \pi$; impossible as $|\sin\theta| \le 1$

Thus, the answer will be $0, \pi$.

SECTION-II

NOTE: Attempt any Three (3) questions.

Q.5.(a) Give the logical proof of De Morgan's Laws. (5)

Ans (i) $(A\cup B)' = A'\cap B'$

Let $x \in (A\cup B)'$

$\Rightarrow x \notin A\cup B$

$\Rightarrow x \notin A$ and $A \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in A'\cap B'$ (1)

But x is an arbitrary member of $(A\cup B)'$

Therefore, (1) means that $(A\cup B)' \subseteq A'\cap B'$ (2)

Now suppose that $y \in A'\cap B'$

$\Rightarrow y \in A'$ and $y \in B'$

$\Rightarrow y \notin A$ and $y \notin B$

$\Rightarrow y \notin A\cup B$

$\Rightarrow y \in (A\cup B)'$

Thus $A'\cap B' \subseteq (A\cup B)'$ (3)

From (2) and (3), we conclude that

$(A\cup B)' = A'\cap B'$

(ii) $(A \cap B)' = A' \cup B'$

It may be proved similarly or deducted from $A \cup B = B \cup A$ by complementation

(iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Let $x \in A \cup (B \cap C)$

$\Rightarrow$ $x \in A$ or $x \in B \cap C$

$\Rightarrow$ If $x \in A$ it must belong to $A \cup B$ and $x \in A \cup C$

$\Rightarrow$ $x \in (A \cup B) \cap (A \cup C)$ (1)

Also if $x \in B \cap C$, then $x \in B$ and $x \in C$.

$\Rightarrow$ $x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow$ $x \in (A \cup B) \cap (A \cup C)$

Thus $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ (2)

Conversely, suppose that

$$y \in (A \cup B) \cap (A \cup C)$$

There are two cases to consider:

$$y \in A, y \notin A$$

In the first case, $y \in A \cup (B \cap C)$

If $y \notin A$, it must belong to B as well as C

i.e., $y \in (B \cap C)$

$\therefore$ $y \in A \cup (B \cap C)$

So in either case,

$$y \in (A \cup B) \cap (A \cup C) \Rightarrow y \in A \cup (B \cap C)$$

Thus $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ (3)

From (2) and (3), it follows that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

It may be proved similarly or deducted from

$$A \cup (B \cup C) = (A \cup B) \cup C$$

by complementation.

(b) Prove that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$

Ans L.H.S $= \begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix}$

Adding C_2 in C_1, we get

$$= \begin{vmatrix} a+b+c & a & a^2 \\ a+b+c & b & b^2 \\ a+b+c & c & c^2 \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

By interchanging rows and columns,

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

By $C_2 - C_1, C_3 - C_1,$

$$= (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix}$$

Expanding by R_1,

$$= (a+b+c) \begin{vmatrix} b-a & c-a \\ b^2-a^2 & c^2-a^2 \end{vmatrix}$$

By taking common: $b-a$ from C_1 and $c-a$ from C_2

$$= (a+b+c)(b-a)(c-a) \begin{vmatrix} 1 & 1 \\ b+a & c+a \end{vmatrix}$$

$$= (a+b+c)(b-a)(c-a)\,[1(c+a) - 1(b+a)]$$

$$= (a+b+c)(b-a)(c-a)(c-b)$$

$$= (a+b+c)(-1)(a-b)(c-a)(-1)(b-c)$$

$$= (a+b+c)(a-b)(b-c)(c-a)$$

$= \text{R.H.S}$ Proved

Q.6.(a) Show that the roots of $(mx+c)^2 = 4ax$ will be equal, if $c = \frac{a}{m}$; $m \neq 0$. (5)

Ans For Answer see Model Paper 1, Q.5.(b).

(b) Resolve into partial fractions of $\frac{2x+1}{(x-1)(x+2)(x+3)}$ (5)

Ans

$$\frac{2x+1}{(x-1)(x+2)(x+3)}$$

Let,

$$\frac{2x+1}{(x-1)(x+2)(x+3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+3} \qquad \text{(i)}$$

Multiply by $(x-1)(x+2)(x+3)$ on both sides

$$2x+1 = A(x+2)(x+3) + B(x-1)(x+3) + C(x-1)(x+2) \quad \text{(ii)}$$

Put $x = 1$ in equation (ii), we have

$$2(1)+1 = A(1+2)(1+3)$$
$$2+1 = A(3)(4)$$
$$\frac{3}{(3)(4)} = A$$
$$\Rightarrow \quad \boxed{A = \frac{1}{4}}$$

Put $x = -2$ in equation (ii), we have

$$2(-2)+1 = B(-2-1)(-2+3) + B(0) + C(0)$$
$$-4+1 = B(-3)(+1)$$
$$-3 = -3B$$
$$\Rightarrow \quad \boxed{B = 1}$$

Put $x = -3$ in equation (ii), we have

$$2(-3)+1 = C(-3-1)(-3+2)$$
$$-6+1 = C(-4)(-1)$$
$$-5 = 4C$$
$$\Rightarrow \quad \boxed{C = \frac{-5}{4}}$$

Putting the values of A, B and C in equation (i), we have

$$\frac{2x+1}{(x-1)(x+2)(x+3)} = \frac{1}{4x-1} + \frac{1}{x+2} - \frac{5}{4(x+3)}$$

Hence partial fractions are

$$\frac{1}{4(x-1)} + \frac{1}{x+2} - \frac{5}{4(x+1)}$$

Q.7.(a) If a, b, c, d are in G.P., prove that $a^2 - b^2, b^2 - c^2, c^2 - a^2$ are in G.P. (5)

Ans If r is the common ratio of the G.P. a, b, c, d

$$r = \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

$$\therefore \quad b = ar \quad \text{(i)}$$
$$c = br = ar^2 \quad \text{(ii)}$$
$$d = cr = ar^3 \quad \text{(iii)}$$

Now $a^2 - b^2, b^2 - c^2, c^2 - d^2$ will be in G.P.

if $\frac{b^2-a^2}{a^2-b^2}=\frac{c^2-d^2}{b^2-c^2}$

or if $(b^2-c^2)^2=(a^2-b^2)(c^2-d^2)$

By using (i) and (ii), we have

$$\text{L.H.S} = (b^2-c^2)^2 = (a^2r^2-a^2r^4)^2$$
$$= a^4r^4(1-r^2)^2$$
$$\text{R.H.S} = (a^2-b^2)(c^2-d^2)$$

By using (i), (ii) and (iii),

$$= (a^2-a^2r^2)(a^2r^4-a^2r^6)$$
$$= a^2(1-r^2)a^2r^4(1-r^2)$$
$$= a^4r^4(1-r^2)^2$$

As L.H.S = R.H.S

So, $a^2-b^2, b^2-c^2, c^2-d^2$ are in G.P.

(b) Find the term involving x^4 in the expansion of $(3-2x)^7$. **(5)**

Ans Let T_{r+1} be the required. Then

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$
$$= \binom{7}{r} 3^{7-r}(-2x)^r$$
$$= \binom{7}{r} 3^{7-r}(-2)^r(x)^r \quad \text{(i)}$$

For the term involving x^4, put exponent of x equal to 4, *i.e.*, r = 4

$$T_{4+1} = \binom{7}{4} 3^{7-4}(-2)^4 x^4$$
$$T_5 = \frac{7\times6\times5}{3\times2\times1}(3^3)(16)x^4$$
$$= 15120\,x^4$$

Q.8.(a) Prove the identity: **(5)**

$$\frac{1}{\operatorname{cosec}\theta-\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\operatorname{cosec}\theta+\cot\theta}$$

Ans

$$\text{L.H.S} = \frac{1}{\operatorname{cosec}\theta-\cot\theta}-\frac{1}{\sin\theta}$$
$$= \frac{\sin\theta-(\operatorname{cosec}\theta-\cot\theta)}{\sin\theta(\operatorname{cosec}\theta-\cot\theta)}$$

$$= \frac{\sin\theta - \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}}{\sin\theta\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)}$$

$$= \frac{\frac{\sin^2\theta - 1 + \cos\theta}{\sin\theta}}{\sin\theta\left(\frac{1-\cos\theta}{\sin\theta}\right)}$$

$$= \frac{\sin^2\theta - 1 + \cos\theta}{\sin\theta\,(1-\cos\theta)}$$

$$= \frac{1 - \cos^2\theta - 1 + \cos\theta}{\sin\theta\,(1-\cos\theta)}$$

$$= \frac{\cos\theta\,(1-\cos\theta)}{\sin\theta\,(1-\cos\theta)}$$

$$\text{L.H.S} = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

Now $\text{R.H.S} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

$$= \frac{\operatorname{cosec}\theta + \cot\theta - \sin\theta}{\sin\theta\,(\operatorname{cosec}\theta + \cot\theta)}$$

$$= \frac{\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} - \sin\theta}{\sin\theta\left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)}$$

$$= \frac{\frac{1 + \cos\theta - \sin^2\theta}{\sin\theta}}{\sin\theta\left(\frac{1+\cos\theta}{\sin\theta}\right)}$$

$$= \frac{1 + \cos\theta - \sin^2\theta}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{\cos\theta + 1 - \sin^2\theta}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{\cos\theta + \cos^2\theta}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{\cos\theta\,(1+\cos\theta)}{\sin\theta\,(1+\cos\theta)}$$

$$R.H.S = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

Hence $\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

(b) Prove the identity $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$. (5)

Ans

$$L.H.S = \frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta}$$

$$= \frac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin(3\theta - \theta)}{\sin\theta\cos\theta}$$

$$= \frac{\sin 2\theta}{\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= 2$$

$$= R.H.S$$

Q.9.(a) Prove that in an equilateral triangle, $r : R : r_1 = 1 : 2 : 3$. (5)

Ans As in equilateral triangle, all sides are equal so we take

$$a = b = c$$

Then $s = \frac{a+a+a}{2}$ $\left(\text{As } s = \frac{a+b+c}{2}\right)$

$$s = \frac{3a}{2}$$

Now $s - a = s - b = s - c$ (As all sides are equal)

$$= \frac{3a}{2} - a = \frac{3a - 2a}{2} = \frac{a}{2}$$

Now $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{\left(\frac{3a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)} = \sqrt{\frac{3a^4}{4\times 4}} = \frac{\sqrt{3}\,a^2}{4}$$

Now $r = \frac{\Delta}{s} = \frac{\sqrt{3}\,a^2}{4} \div \frac{3a}{2}$ $\left(\text{As } s = \frac{3a}{2}\right)$

$$r = \frac{\sqrt{3}\,a^2}{4} \times \frac{2}{3a} = \frac{\sqrt{3}\,a}{6} = \frac{\sqrt{3}\,a}{3\times 2} = \frac{a}{2\sqrt{3}}$$

$\Rightarrow \quad \boxed{r = \frac{a}{2\sqrt{3}}}$

Now $\quad R = \frac{abc}{4\Delta} = \frac{(a)(a)(a)}{4\frac{\sqrt{3}\,a^2}{4}} = \frac{a^3}{\sqrt{3}\,a^2}$

$$\boxed{R = \frac{a}{\sqrt{3}}}$$

As $\quad r_1 = \frac{\Delta}{s-a} = \frac{\sqrt{3}\,a^2}{4} \div \left(\frac{a}{2}\right)$

$$= \frac{\sqrt{3}\,a^2}{4} \times \frac{2}{a} = \frac{\sqrt{3}\,a}{2}$$

$\Rightarrow \quad \boxed{r_1 = \frac{\sqrt{3}\,a}{2}}$

Now L.H.S $= r : R : r_1$

Putting values of r, R and r_1

$$= \frac{a}{2\sqrt{3}} : \frac{a}{\sqrt{3}} : \frac{\sqrt{3}\,a}{2}$$

$\div$ by a

$$= \frac{1}{2\sqrt{3}} : \frac{1}{\sqrt{3}} : \frac{\sqrt{3}}{2}$$

Multiplying by $\sqrt{3} \times 2$

$$= 2\sqrt{3} \times \frac{1}{2\sqrt{3}} : \sqrt{3} \times 2 \times \frac{1}{\sqrt{3}} : \sqrt{3} \times 2 \times \frac{\sqrt{3}}{2}$$

$$= 1 : 2 : \sqrt{9} = 1 : 2 : 3$$

So proved $\boxed{r : R : r_1 = 1 : 2 : 3}$

(b) Prove that $\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1} 3 = \frac{\pi}{4}$. **(5)**

Ans Let

$$x = \sin^{-1}\frac{1}{\sqrt{5}}$$

$$\sin x = \frac{1}{\sqrt{5}}$$

$$\cos x = \sqrt{1 - \sin^2 x}$$

$$= \sqrt{1 - \left(\frac{1}{\sqrt{5}}\right)^2}$$

$$\cos x = \sqrt{\frac{5-1}{5}}$$

$$\cos x = \frac{2}{\sqrt{5}}$$

$$y = \cot^{-1} 3$$

$$\cot y = 3$$

$$\operatorname{cosec} y = \sqrt{1 + \cot^2 y}$$

$$= \sqrt{1 + (3)^2}$$

$$\operatorname{cosec} y = \sqrt{10}$$

$$\sin y = \frac{1}{\sqrt{10}}$$

$$\cos y = \sqrt{1 - \sin^2 y}$$

$$= \sqrt{1 - \left(\frac{1}{10}\right)^2}$$

$$= \sqrt{\frac{10-1}{10}}$$

$$\cos y = \frac{3}{\sqrt{10}}$$

Using

$$\sin(x + y) = \sin x \,.\, \cos y + \cos x \,.\, \sin y$$

$$= \frac{1}{\sqrt{5}} \cdot \frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{10}}$$

$$= \frac{3+2}{\sqrt{50}}$$

$$= \frac{5}{5\sqrt{2}}$$

$$x + y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1} 3 = \frac{\pi}{4}$$