

10th Class 2017

Math (Science)	Group-I	PAPER-II
Time: 20 Minutes	(Objective Type)	Max. Marks: 15

Note: Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1- An equation which remains unchanged when x is replaced by $\frac{1}{x}$ is called a / an:

- (a) Exponential equation
- (b) Reciprocal equation ✓
- (c) Radical equation
- (d) Quadratic equation

2- If α, β are the roots of $3x^2 + 5x - 2 = 0$, then $\alpha + \beta$ is:

- (a) $\frac{5}{3}$
- (b) $\frac{3}{5}$
- (c) $-\frac{5}{3}$ ✓
- (d) $-\frac{2}{3}$

3- Cube roots of -1 are:

- (a) $-1, -\omega, -\omega^2$ ✓
- (b) $-1, \omega, -\omega^2$
- (c) $-1, -\omega, \omega^2$
- (d) $1, -\omega, -\omega^2$

4- The fourth proportional w of $x : y :: v : w$ is:

- (a) $\frac{xy}{v}$
- (b) $\frac{vy}{x}$ ✓
- (c) xyv
- (d) $\frac{x}{vy}$

5- If $a : b = x : y$, then alternando property is:

- (a) $\frac{a}{x} = \frac{b}{y}$ ✓
- (b) $\frac{a}{b} = \frac{x}{y}$
- (c) $\frac{a+b}{b} = \frac{x+y}{y}$
- (d) $\frac{a-b}{x} = \frac{x-y}{y}$

10th Class 2017

Math (Science)	Group-I	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: 12

(i) Write the name of any two methods for solving a quadratic equation.

Ans The name of any two methods for solving a quadratic equation are:

1. Factorization Method.
2. Completing Square Method.

(ii) Solve: $x^2 + 2x - 2 = 0$ **Ans** Here, $a = 1$, $b = 2$, $c = -2$

We may solve the above equation through quadratic formula, so

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)} \\
 &= \frac{-2 \pm \sqrt{4 + 8}}{2} \\
 &= \frac{-2 \pm \sqrt{12}}{2} \\
 &= \frac{-2 \pm 2\sqrt{3}}{2} \\
 &= -1 \pm \sqrt{3}
 \end{aligned}$$

(iii) Evaluate : $(1 - 3w - 3w^2)^5$ **Ans** Given:

$$(1 - 3w - 3w^2)^5$$

By taking common, we get

$$= [1 - 3(w + w^2)]^5$$

- 6- $\frac{3\pi}{4}$ radian = ---- :
- (a) 115° (b) 150°
(c) 30° (d) 135° ✓
- 7- The set having only one element is called:
- (a) Null set (b) Power set
(c) Singleton set ✓ (d) Subset
- 8- If $A \subseteq B$, then $A - B$ is equal to:
- (a) A (b) B
(c) $B - A$ (d) ϕ ✓
- 9- A frequency polygon is a many sided ---:
- (a) Closed figure ✓ (b) Rectangle
(c) Circle (d) Square
- 10- $\sec^2 \theta = - - -$:
- (a) $1 - \sin^2 \theta$ (b) $1 + \tan^2 \theta$ ✓
(c) $1 + \cos^2 \theta$ (d) $1 - \tan^2 \theta$
- 11- Radii of a circle are:
- (a) Double of the diameter
(b) All unequal
(c) Half of any chord (d) All equal ✓
- 12- A tangent line intersects the circle at ---- :
- (a) Three points (b) Two points
(c) Single point ✓ (d) Four points
- 13- The arcs opposite to incongruent central angles of a circle are always ---- :
- (a) Congruent (b) Parallel
(c) Perpendicular (d) Incongruent ✓
- 14- The length of the diameter of a circle is how many times the radius of the circle:
- (a) 1 (b) 2 ✓
(c) 3 (d) 4
- 15- The tangent and radius of a circle at the point of contact are ---- :
- (a) Parallel (b) Not perpendicular
(c) Perpendicular ✓ (d) Not parallel

$$w + w^2 = -1$$

As we know that:

$$\begin{aligned} &= [1 - 3(-1)]^5 \\ &= (1 + 3)^5 \\ &= 4^5 \\ &= 1024 \end{aligned}$$

(iv) Evaluate: $\omega^{37} + \omega^{38} - 5$

Ans Given:

$$\begin{aligned} &\omega^{37} + \omega^{38} - 5 \\ &= \omega^{36} \cdot \omega + \omega^{36} \cdot \omega^2 - 5 \\ &= \omega^{36}(\omega + \omega^2) - 5 \\ &= (\omega^3)^{12}(-1) - 5 \\ &= -1 - 5 \\ &= -6 \end{aligned}$$

(v) Without solving find the sum and the product of roots of quadratic equation: $3x^2 + 7x - 11 = 0$

Ans Here, $a = 3$, $b = 7$, $c = -11$

Sum of the roots:

$$S = \alpha + \beta = \frac{-b}{a} = \frac{-7}{3}$$

Product of the roots:

$$P = \alpha\beta = \frac{c}{a} = \frac{-11}{3}$$

(vi) Write the quadratic equation having the roots: $-1, -7$

Ans Sum of the roots:

$$\alpha + \beta = -1 + (-7) = -8$$

Product of the roots:

$$\alpha\beta = (-1)(-7) = 7$$

Thus the quadratic equation will be:

$$x^2 - Sx + P = 0$$

$$x^2 - (-8)x + 7 = 0$$

$$x^2 + 8x + 7 = 0$$

(vii) Define direct variation.

Ans If two quantities are related in such a way that increase (decrease) in one quantity causes increase

(decrease) in the other quantity then this variation is called direct variation.

(viii) Find the fourth proportional to 8, 7, 6.

Ans Let the fourth proportional is x :

$$8 : 7 :: 6 : x$$

$$8 \times x = 7 \times 6$$

$$x = \frac{42}{8}$$

$$x = \frac{21}{4}$$

(ix) Find x if $6 : x :: 3 : 5$.

Ans $x \times 3 = 6 \times 5$

$$x = \frac{6 \times 5}{3}$$

$$x = \frac{30}{3}$$

$$\boxed{x = 10}$$

3. Write short answers to any SIX (6) questions: 12

(i) Define a rational fraction.

Ans An expression of the form $\frac{N(x)}{D(x)}$, where $N(x)$ and $D(x)$ are polynomials in x with real coefficients and $D(x) \neq 0$, is called a rational fraction.

(ii) Resolve $\frac{1}{x^2 - 1}$ into partial fraction.

Ans
$$\frac{1}{x^2 - 1} = \frac{1}{(x + 1)(x - 1)}$$

$$\frac{1}{x^2 - 1} = \frac{A}{x + 1} + \frac{B}{x - 1}$$

$$1 = \frac{A}{(x + 1)}(x^2 - 1) + \frac{B}{(x - 1)}(x^2 - 1)$$

$$1 = A(x - 1) + B(x + 1) \quad (i)$$

Put $x = 1$ in (i)

$$1 = A(1 - 1) + B(1 + 1)$$

$$1 = 0 + 2B$$

$$2B = 1$$

$$\boxed{B = \frac{1}{2}}$$

Similarly, put $x = -1$ in (i),

$$1 = A(-1 - 1) + B(-1 + 1)$$

$$1 = A(-2) + 0$$

$$1 = -2A$$

$$\Rightarrow -2A = 1$$

$$\boxed{A = -\frac{1}{2}}$$

Finally, by putting the values in (i), we have

$$\frac{1}{x^2 - 1} = \frac{1}{2(x - 1)} - \frac{1}{2(x + 1)}$$

(iii) Define subset.

Ans If A and B are two sets and every element of A is a member of B then A is called subset of B.

(iv) If $L = \{a, b, c\}$, $M = \{3, 4\}$ then find $L \times M$.

Ans $L \times M = \{a, b, c\} \times \{3, 4\}$

$$L \times M = \{(a, 3), (a, 4), (b, 3), (b, 4), (c, 3), (c, 4)\}$$

(v) Find domain and range of the binary relation,
 $R = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$.

Ans Dom $R = \{1, 2, 3, 4\}$

Range $R = \{1, 2, 3, 4\}$

(vi) If $(2a + 5, 3) = (7, b - 4)$, find a, b.

Ans By comparing the values, we get

$$2a + 5 = 7 \quad ; \quad 3 = b - 4$$

$$2a = 7 - 5 \quad ; \quad 3 + 4 = b$$

$$2a = 2 \quad ; \quad 7 = b$$

$$a = \frac{2}{2}$$

$$; \Rightarrow \boxed{b = 7}$$

$$\boxed{a = 1}$$

Thus, $\{a = 1, b = 7\}$

Write two properties of arithmetic mean.

(vii) **Ans** Two properties of arithmetic mean are:

1. Mean is affected by change in origin.
2. Sum of the deviations of the variable X from its mean is always zero.

(viii) **Define mode.**

Ans Mode is defined as the most frequent value in the data.

(ix) The sugar contents for a random sample of 6 packs of juices of a certain brand are found to be 2.3, 2.7, 2.5, 2.9, 3.1 and 1.9 milligram, find the median.

Ans Arranging the values by increasing order 1.9, 2.3, 2.5, 2.7, 2.9, 3.1.

$$\begin{aligned}\text{Median} &= \frac{1}{2} [\text{size of } (3^{\text{rd}} + 4^{\text{th}}) \text{ values}] \\ &= \frac{2.5 + 2.7}{2} \\ &= 2.6 \text{ Milligram}\end{aligned}$$

4. Write short answers to any SIX (6) questions: 12

(i) **Define radian measure of an angle.**

Ans The angle subtended at the centre of the circle by an arc, whose length is equal to the radius of the circle is called one Radian.

(ii) **Convert 15° to radian.**

$$\begin{aligned}\text{Ans} \quad 15^\circ &= 15 \times \frac{\pi}{180} \text{ radian} \\ &= \frac{\pi}{12} \text{ radian}\end{aligned}$$

(iii) **Find 'r', when $l = 56 \text{ cm}$, $\theta = 45^\circ$.**

Ans $l = 56 \text{ cm}$, $\theta = 45^\circ$, $r = ?$

By converting the θ into radians

$$45^\circ = 45 \times \frac{\pi}{180} \text{ radian}$$

$$= \frac{\pi}{4} \text{ radians}$$

We have,

$$l = r \theta$$

$$\Rightarrow r = \frac{l}{\theta}$$

$$= \frac{56}{\frac{\pi}{4}}$$

$$= \frac{56 \times 4}{\pi}$$

$$r = 71.27 \text{ cm}$$

(iv) What is meant by zero dimension?

Ans Projection of a vertical line segment \overline{CD} on a line segment \overline{AB} is a point on \overline{AB} which is of zero dimension.

(v) Define chord of a circle.

Ans The joining of any two points on the circumference of the circle is called chord of a circle.

(vi) Define tangent to a circle.

Ans A tangent to a circle is the straight line which touches the circumference at one point only.

(vii) What is meant by sector of a circle?

Ans The sector of a circle is an area bounded by any two radii and the arc intercepted between them.

(viii) Define circumangle.

Ans A circumangle is subtended between any two chords of a circle, having common point on its circumference.

(ix) Define inscribed circle.

Ans A circle which touches the three sides of a triangle internally is known as inscribed circle.

(Part-II)

NOTE: Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve the equation by completing square: (4)

$$11x^2 - 34x + 3 = 0$$

Ans

$$11x^2 - 34x = -3$$

$$x^2 - \frac{34}{11}x = \frac{-3}{11}$$

Adding $\left(\frac{17}{11}\right)^2$ on both sides

$$x^2 - 2(x) \left(\frac{17}{11}\right) + \left(\frac{17}{11}\right)^2 = \frac{-3}{11} + \left(\frac{17}{11}\right)^2$$

$$\begin{aligned} \left(x - \frac{17}{11}\right)^2 &= \frac{-3}{11} + \frac{289}{121} \\ &= \frac{-33 + 289}{121} \\ &= \frac{256}{121} \end{aligned}$$

Taking square root on both sides, we have

$$x - \frac{17}{11} = \pm \frac{16}{11}$$

$$x = \frac{17}{11} \pm \frac{16}{11}$$

$$x = \frac{17}{11} + \frac{16}{11}$$

$$= \frac{17 + 16}{11}$$

$$= \frac{33}{11}$$

$$x = 3$$

$$x = \frac{17}{11} - \frac{16}{11}$$

$$= \frac{17 - 16}{11}$$

$$x = \frac{1}{11}$$

- (b) If α, β are the roots of equation $lx^2 + mx + n = 0$,
 ($l \neq 0$) then find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$. (4)

Ans $a = l, b = m, c = n$

$$\alpha + \beta = \frac{-b}{a} = \frac{-m}{l}$$

$$\alpha\beta = \frac{c}{a} = \frac{n}{l}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{\left(\frac{-m}{l}\right)^2 - 2\left(\frac{n}{l}\right)}{\left(\frac{n}{l}\right)^2}$$

$$= \frac{\frac{m^2}{l^2} - \frac{2n}{l}}{\frac{n^2}{l^2}}$$

$$= \frac{\frac{m^2 - 2ln}{l^2}}{\frac{n^2}{l^2}}$$

$$= \frac{m^2 - 2ln}{n^2}$$

$$= \frac{1}{n^2} (m^2 - 2ln)$$

- Q.6.(a) Using theorem of componendo-dividendo find
 the value of: $\frac{x+2y}{x-2y} + \frac{x+2z}{x-2z}$ if $x = \frac{4yz}{y+z}$. (4)

Ans

$$x = \frac{4yz}{y+z}$$

$$\frac{x}{2y} = \frac{2z}{y+z}$$

$$\frac{x + 2y}{x - 2y} = \frac{2z + y + z}{2z - y - z}$$

$$\frac{x + 2y}{x - 2y} = \frac{y + 3z}{z - y}$$

(1)

Similarly,

$$\frac{x}{2z} = \frac{2y}{y + z}$$

$$\frac{x + 2z}{x - 2z} = \frac{2y + y + z}{2y - y - z}$$

$$= \frac{3y + z}{y - z}$$

$$= -\left(\frac{3y + z}{z - y}\right)$$

$$\frac{x + 2z}{x - 2z} = \frac{-3y - z}{z - y}$$

(2)

From (1) and (2), we have

$$\frac{x + 2y}{x - 2y} + \frac{x + 2z}{x - 2z} = \frac{y + 3z}{z - y} + \frac{-3y - z}{z - y}$$

$$= \frac{y + 3z - 3y - z}{z - y}$$

$$= \frac{2z - 2y}{z - y}$$

$$= \frac{2(z - y)}{z - y}$$

$$= 2$$

(b) Resolve into partial fractions: $\frac{x - 11}{(x - 4)(x + 3)}$ (4)

Ans

$$\frac{x - 11}{(x - 4)(x + 3)} = \frac{A}{x - 4} + \frac{B}{x + 3}$$

$$x - 11 = A(x + 3) + B(x - 4) \quad (i)$$

Put $x = 4$, $x = -3$ in (i)

$$\text{Firstly, } 4 - 11 = A(4 + 3) + B(4 - 4)$$

$$-7 = A(7) + 0$$

$$\Rightarrow 7A = -7$$

$$\boxed{A = -1}$$

And

$$\begin{aligned} -3 - 11 &= A(-3 + 3) + B(-3 - 4) \\ -14 &= 0 + B(-7) \end{aligned}$$

$$\Rightarrow -7B = -14$$

$$\boxed{B = 2}$$

So,

$$\frac{x - 11}{(x - 4)(x + 3)} = \frac{-1}{x - 4} + \frac{2}{x + 3}$$

Q.7.(a) If $U = \{1, 2, 3, \dots, 10\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{1, 4, 7, 10\}$ then verify that $A - B = A \cap B'$. (4)

Ans

$$\text{L.H.S} = A - B$$

$$= \{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}$$

$$= \{3, 5, 9\}$$

(i)

$$B' = U - B$$

$$= \{1, 2, 3, \dots, 10\} - \{1, 4, 7, 10\}$$

$$= \{2, 3, 5, 6, 8, 9\}$$

$$\text{R.H.S} = A \cap B'$$

$$= \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 6, 8, 9\}$$

$$= \{3, 5, 9\}$$

(ii)

So, proved from (i) and (ii),

$$\text{L.H.S} = \text{R.H.S}$$

$$A - B = A \cap B'$$

(b) Calculate the variance for the data: (4)

10, 8, 9, 7, 5, 12, 8, 6, 8, 2

Ans

X	X ²
10	100
8	64
9	81
7	49
5	25
12	144
8	64

6	36
8	64
2	4
<hr/>	
75	631

Here, $\Sigma X = 75$, $\Sigma X^2 = 631$, $n = 10$

$$\begin{aligned}\text{Variance} = S^2 &= \frac{\Sigma X^2}{n} - \left(\frac{\Sigma X}{n}\right)^2 \\ &= \frac{631}{10} - \left(\frac{75}{10}\right)^2 \\ &= 63.1 - 56.25 \\ S^2 &= 6.85\end{aligned}$$

Q.8.(a) Prove that: $\sin \theta (\tan \theta + \cot \theta) = \sec \theta$ (4)

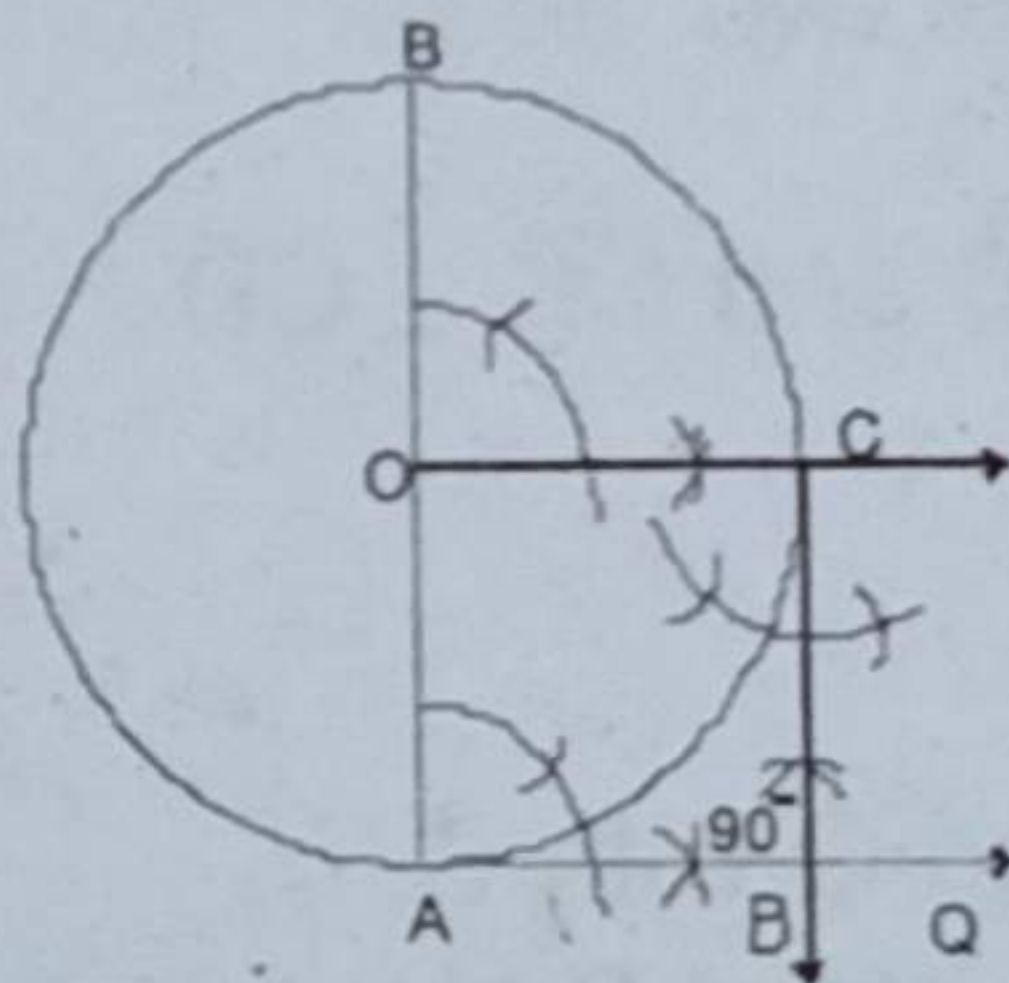
Ans L.H.S = $\sin \theta (\tan \theta + \cot \theta)$

$$\begin{aligned}&= \sin \theta \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right) \\ &= \sin \theta \left(\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \right) \\ &= \sin \theta \left(\frac{1}{\cos \theta \sin \theta} \right) \\ &= \frac{1}{\cos \theta} \\ &= \sec \theta \\ &= \text{R.H.S}\end{aligned}$$

Proved

(b) Draw two perpendicular tangents to a circle of radius 3 cm.

Ans



Step of Construction:**Steps:**

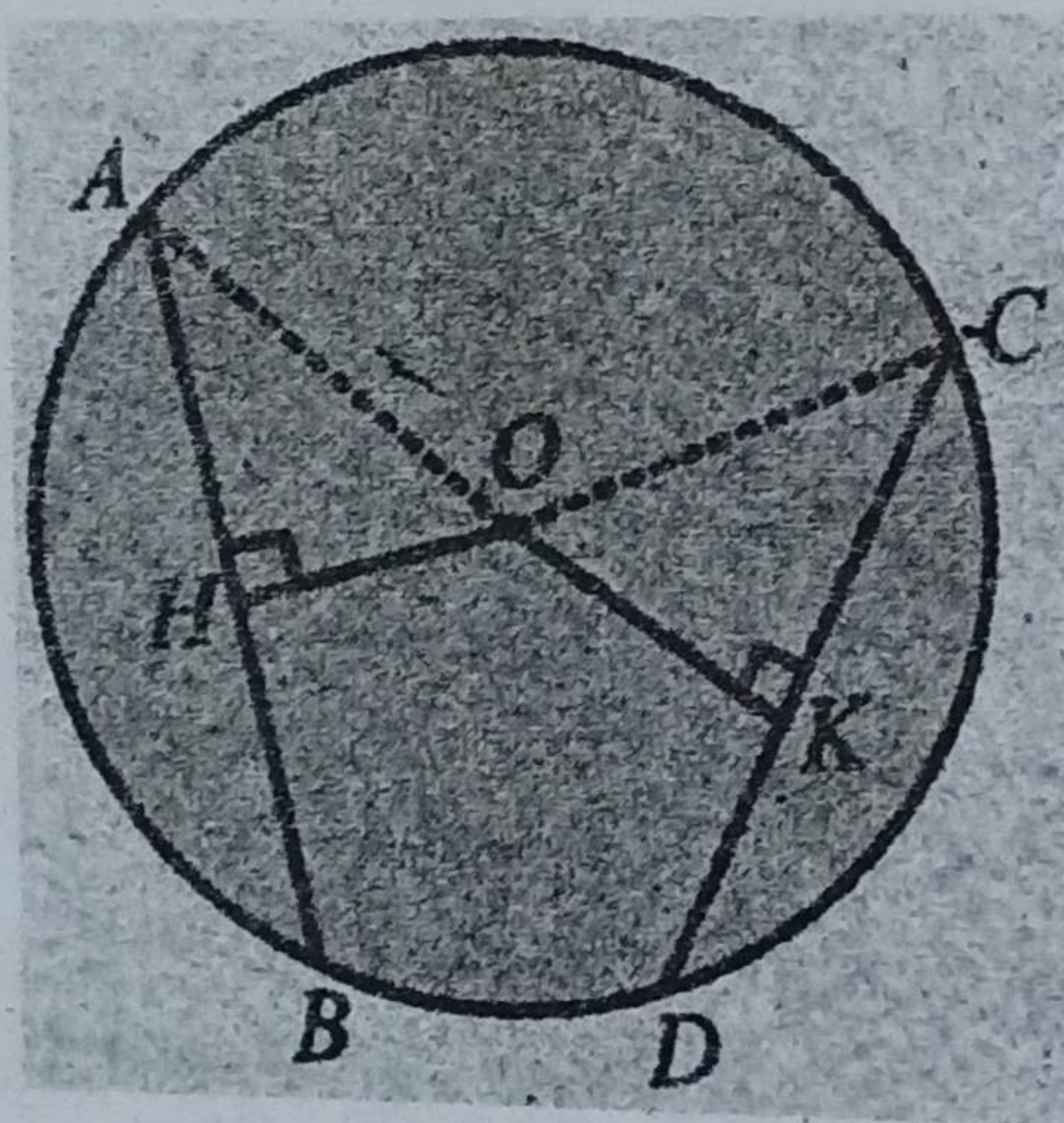
1. Take a point O.
2. Take O as centre and draw circle of radius 3 cm.
3. Take AOB any diameter of the circle.
4. Draw $m\angle BOC = 90^\circ$, $m\angle AOC = 90^\circ$
5. Draw tangents at point A, C. These are \vec{CP} , \vec{AQ} .

Result:

\vec{AQ} , \vec{CP} are required tangents at point D at 90° .

Q.9. Prove that if two chords of a circle are congruent then they will be equidistant from the centre. (4)

Ans

**Given:**

\overline{AB} and \overline{CD} are two equal chords of a circle with centre at O.

So that $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$.

To prove:

$$m\overline{OH} = m\overline{OK}$$

Construction:

Join O with A and O with C.

So that we have $\triangle OAH$ and $\triangle OCK$.

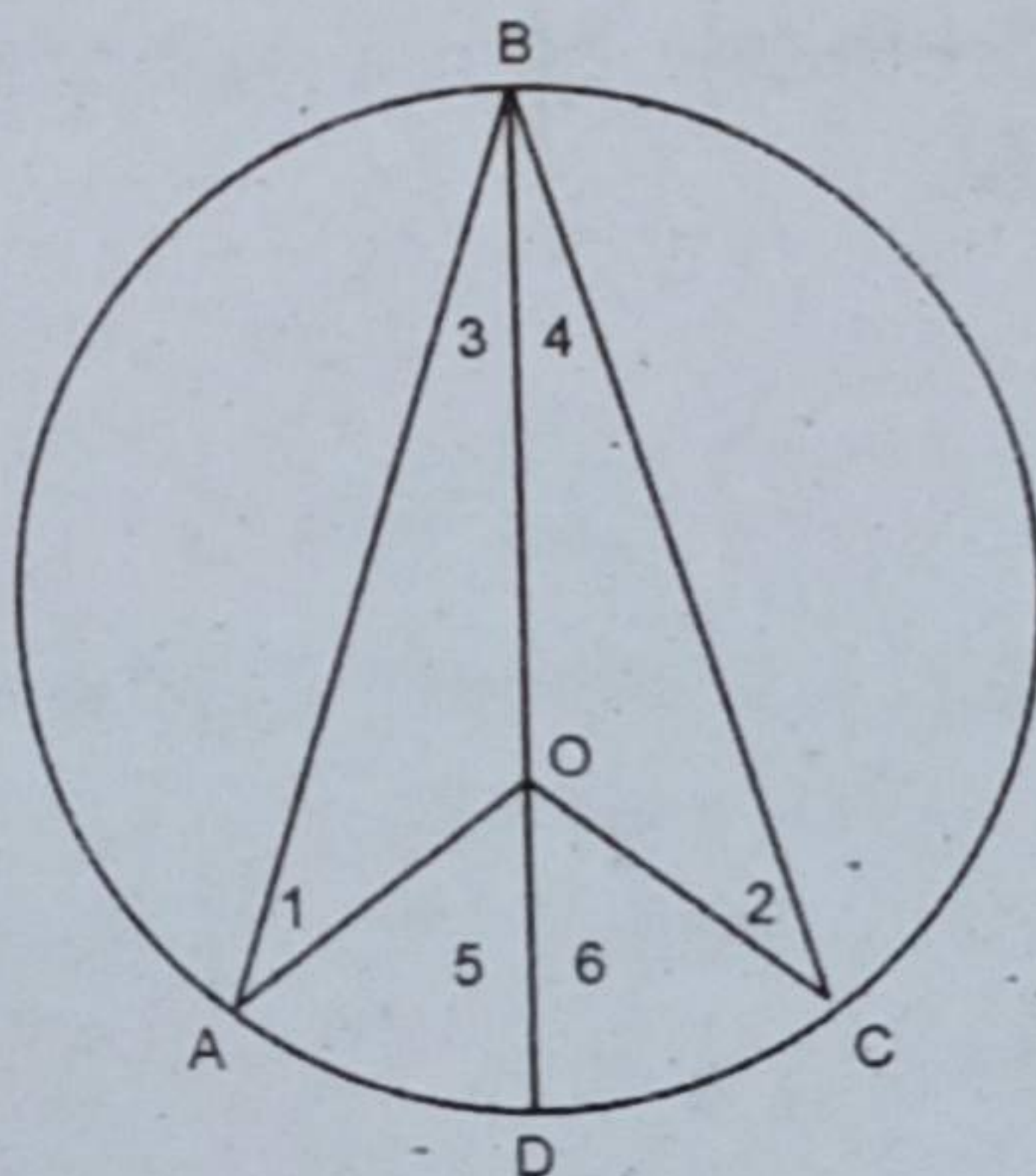
Proof:

Statements	Reasons
\overline{OH} bisects chord \overline{AB} i.e., $m\overline{AH} = \frac{1}{2} m\overline{AB}$ (i)	$\overline{OH} \perp \overline{AB}$ By Theorem 3
Similarly \overline{OK} bisects chord \overline{CD} i.e., $m\overline{CK} = \frac{1}{2} m\overline{CD}$ (ii)	$\overline{OK} \perp \overline{CD}$ By Theorem 3
But $m\overline{AB} = m\overline{CD}$ (iii)	Given
Hence $m\overline{AH} = m\overline{CK}$ (iv)	Using (i), (ii) & (iii)
Now in $\angle rt \Delta^s OAH \leftrightarrow OCK$ hyp $\overline{OA} = \text{hyp } \overline{OC}$ $m\overline{AH} = \overline{CK}$ $\therefore \Delta OAH \cong \Delta OCK$ $\Rightarrow m\overline{OH} = m\overline{OK}$	Given $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$ Radii of the same circle Already proved in (iv) H.S postulate

OR

Prove that the measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.

Ans



Given:

\widehat{AC} is an arc of a circle with center O. Whereas $\angle AOC$ is the central angle and $\angle ABC$ is circumangle.

To prove:

$$m\angle AOC = 2m\angle ABC$$

Construction:

Join B with O and produce it to meet the circle at D.

Write angles $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$ and $\angle 6$ as shown in the figure.

Proof:

Statements	Reasons
As $m\angle 1 = m\angle 3$ (i)	Angles opposite to equal sides in $\triangle OAB$.
and $m\angle 2 = m\angle 4$ (ii)	Angles opposite to equal sides in $\triangle OBC$.
Now $m\angle 5 = m\angle 1 + m\angle 3$ (iii)	External angle is the sum of internal opposite angles.
Similarly,	
$m\angle 6 = m\angle 2 + m\angle 4$ (iv)	
Again	
$m\angle 5 = m\angle 3 + m\angle 3 = 2m\angle 3$ (v)	Using (i) and (iii)
And	
$m\angle 6 = m\angle 4 + m\angle 4 = 2m\angle 4$ (vi)	Using (ii) and (iv)
$\Rightarrow m\angle 5 + m\angle 6 = 2m\angle 3 + m\angle 4$	Adding (v) and (vi)
$\Rightarrow m\angle AOC = 2(m\angle 3 + m\angle 4)$ $= 2m\angle ABC$	