

## 10th Class 2019

Math (Science)	Group-II	PAPER-II
Time: 20 Minutes	(Objective Type)	Max. Marks: 15

**Note:** Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

- 1-1- A fraction in which the degree of the numerator is less than the degree of the denominator is called ----:
- (a) An equation (b) An improper fraction  
(c) An identity (d) A proper fraction ✓
- 2- A circle has only one ----:
- (a) Secant (b) Chord  
(c) Diameter (d) Centre ✓
- 3-  $\frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta}$  :
- (a)  $2 \sec^2 \theta$  ✓ (b)  $2 \cos^2 \theta$   
(c)  $\sec^2 \theta$  (d)  $\cos \theta$
- 4- How many common tangents can be drawn for two touching circles:
- (a) 1 (b) 2  
(c) 3 ✓ (d) 4
- 5- The number of methods to solve a quadratic equation is:
- (a) 1 (b) 2  
(c) 3 ✓ (d) 4
- 6- Power set of an empty set is:
- (a)  $\phi$  (b)  $\{a\}$   
(c)  $\{\phi, \{a\}\}$  (d)  $\{\phi\}$  ✓
- 7- The symbol for a triangle is denoted by:
- (a)  $\angle$  (b)  $\Delta$  ✓  
(c)  $\perp$  (d)  $\odot$



- 8- In a ratio  $a : b$ ,  $a$  is called:  
(a) Relation (b) Antecedent ✓  
(c) Consequent (d) Proportion
- 9- The length of a chord and the radial segment of a circle are congruent, then the central angle made by the chord will be ----:  
(a)  $30^\circ$  (b)  $45^\circ$   
(c)  $60^\circ$  ✓ (d)  $75^\circ$
- 10- If  $A$  and  $B$  are disjoint sets, then  $A \cup B$  is equal to:  
(a)  $A$  (b)  $B$   
(c)  $\phi$  (d)  $B \cup A$  ✓
- 11- Two square roots of unity are:  
(a)  $1, -1$  ✓ (b)  $1, \omega$   
(c)  $1, -\omega$  (d)  $\omega, \omega^2$
- 12- The third proportional of  $x^2$  and  $y^2$  is:  
(a)  $\frac{y^2}{x^2}$  (b)  $x^2y^2$   
(c)  $\frac{y^4}{x^2}$  ✓ (d)  $\frac{y^2}{x^4}$
- 13- A histogram is a set of adjacent:  
(a) Squares (b) Rectangles ✓  
(c) Circles (d) Triangles
- 14- Sum of cube roots of unity is:  
(a)  $0$  ✓ (b)  $1$   
(c)  $-1$  (d)  $3$
- 15-  $\frac{3\pi}{4}$  radians = :  
(a)  $115^\circ$  (b)  $135^\circ$  ✓  
(c)  $150^\circ$  (d)  $30^\circ$



## 10th Class 2019

Math (Science)	Group-II	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: 12

(i) Define reciprocal equation.

**Ans** An equation is said to be a reciprocal equation, if it remains unchanged, when  $x$  is replaced by  $\frac{1}{x}$ .

(ii) Solve by factorization:  $5x^2 = 15x$ **Ans** Given:  $5x^2 = 15x$ 

$$5x^2 - 15x = 0$$

$$5x(x - 3) = 0$$

$$5x = 0 \quad ; \quad x - 3 = 0$$

$$x = 0 \quad ; \quad x = 3$$

So, the solution set =  $\{0, 3\}$ .

(iii) Find discriminant of the quadratic equation:

$$4x^2 - 7x - 2 = 0$$

**Ans** Here:  $a = 4, b = -7, c = -2$ 

$$\text{Discriminant} = b^2 - 4ac$$

$$= (-7)^2 - 4(4)(-2)$$

$$= 49 + 32$$

$$= 81$$

(iv) Evaluate:  $(9 + 4\omega + 4\omega^2)^3$ **Ans** Given:  $(9 + 4\omega + 4\omega^2)^3$ 

$$= [9 + 4(\omega + \omega^2)]^3$$

$$= [9 + 4(-1)]^3 \quad \because \omega + \omega^2 = -1$$

$$= (9 - 4)^3$$

$$= 5^3 = 125$$

(v) Write the quadratic equation having roots 4, 9.

**Ans** As 4 and 9 are the roots of the required quadratic equation, so



Sum of roots:  $S = 4 + 9 = 13$

Product of roots:  $P = 4(9) = 36$

General quadratic equation, having roots, is

$$x^2 - Sx + P = 0 \quad (i)$$

By putting the values in (i), we get the required quadratic equation, as:

$$x^2 - 13x + 36 = 0$$

(vi) Using synthetic division, divide  $p(x) = x^4 - x^2 + 15$  by  $x + 1$ .

**Ans**  $(x^4 - x^2 + 15) \div (x + 1)$

As  $x + 1 = x - (-1)$ ,

So,  $a = -1$

Now, write the coefficients of dividend in a row and  $a = -1$  on the left side.

$$\begin{array}{r|rrrrr} & 1 & 0 & -1 & 0 & 15 \\ -1 & \downarrow & -1 & 1 & 0 & 0 \\ \hline & 1 & -1 & 0 & 0 & 15 \end{array}$$

$$\therefore \text{Quotient} = Q(x) = x^3 - x^2 + 0.x + 0$$

$$Q(x) = x^3 - x^2$$

and Remainder = 15

(vii) If  $3(4x - 5y) = 2x - 7y$ , find the ratio  $x : y$ .

**Ans**  $3(4x - 5y) = 2x - 7y$

$$12x - 15y = 2x - 7y$$

$$12x - 2x = -7y + 15y$$

$$10x = 8y$$

$$\frac{x}{y} = \frac{8}{10}$$

$$\frac{x}{y} = \frac{4}{5}$$

By converting the above fraction into ratio, we get

$$x : y = 4 : 5$$

(viii) Find the fourth proportional to: 8, 7, 6.

**Ans** Let  $x$  be the fourth proportional, then

$$8 : 7 :: 6 : x$$



Product of extremes = Product of means

$$8(x) = 7(6)$$

$$x = \frac{42}{8}$$

Hence, Fourth Proportional:

$$x = \frac{21}{4}$$

(ix) Define joint variation.

**Ans** A combination of direct and inverse variations of one or more than one variables forms joint variation.

3. Write short answers to any SIX (6) questions: 12

(i) Define fraction.

**Ans** A fraction is an indicated quotient of two numbers or algebraic expressions.

(ii) Define De-Morgan's laws.

**Ans** For any two sets A and B, De-Morgan's laws are:

$$1. (A \cup B)' = A' \cap B'$$

$$2. (A \cap B)' = A' \cup B'$$

(iii) If  $A = \{1, 3, 5, 7, 9\}$  and  $B = \{1, 4, 7, 10\}$ , then find  $(A - B)$ .

**Ans**  $A - B = \{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}$   
 $= \{3, 5, 9\}$

(iv) If  $A = \{a, b\}$  and  $B = \{c, d\}$ , then find  $A \times B$  and  $B \times A$ .

**Ans** Given,  $A = \{a, b\}$  and  $B = \{c, d\}$

$$A \times B = \{a, b\} \times \{c, d\}$$

$$= \{(a, c), (a, d), (b, c), (b, d)\}$$

$$B \times A = \{c, d\} \times \{a, b\}$$

$$= \{(c, a), (c, b), (d, a), (d, b)\}$$

(v) Find domain and the range of  $R = \{(1, 1), (2, 3), (3, 4), (4, 3), (5, 4)\}$ .

**Ans** From above function:

$$\text{Dom } R = \{1, 2, 3, 4, 5\}$$

$$\text{Range } R = \{1, 3, 4\}$$

(vi) Define arithmetic mean and give an example.

**Ans** Arithmetic Mean:



Arithmetic Mean (or simply called Mean) is a measure that determines a value (observation) of the variable under study by dividing the sum of all values (observations) of the variable by their number of observations. In symbols,

$$\bar{X} = \frac{\sum x}{n} \quad (\text{For ungrouped data})$$

$$\bar{X} = \frac{\sum fx}{\sum f} \quad (\text{Grouped data})$$

**Example:**

Marks of each student = 45, 60, 74, 58, 65, 63, 49

No. of values =  $n = 7$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{45 + 60 + 74 + 58 + 65 + 63 + 49}{7}$$

$$\bar{x} = \frac{414}{7}$$

$$\bar{x} = 59.14 \text{ marks}$$

- (vii) Find range for the weights of students: 110, 109, 84, 89, 77, 104, 74, 97, 49, 59, 103, 62.

**Ans** Maximum value =  $X_m = 110$

Minimum value =  $X_0 = 49$

$$\begin{aligned} \text{So, Range} &= X_m - X_0 \\ &= 110 - 49 \\ &= 61 \end{aligned}$$

- (viii) On 5 terms test in mathematics, a student has made marks of 82, 93, 86, 92 and 79. Find the median for the marks.

**Ans** By arranging the marks in ascending order, the arranged data is:

79, 82, 86, 92, 93

Since number of observations is odd, i.e.,  $n = 5$ .



Median =  $\tilde{x}$  = size of  $\left(\frac{n+1}{2}\right)$ th observation

$\tilde{x}$  = size of  $\left(\frac{5+1}{2}\right)$ th observation

$\tilde{x}$  = size of 3<sup>rd</sup> observation

$\tilde{x} = 86$

(ix) For the following data, find the harmonic mean:

x	12	5	8	4
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**Ans**

x	$\frac{1}{x}$
12	0.0833
5	0.2
8	0.125
4	0.25
SUM	0.6583

$$\begin{aligned}\text{Harmonic Mean} = \text{H.M} &= \frac{n}{\sum\left(\frac{1}{x}\right)} \\ &= \frac{4}{0.6583} \\ &= 6.0763\end{aligned}$$

#### 4. Write short answers to any SIX (6) questions: 12

(i) Define an angle.

**Ans** An angle is defined as the union of two non-collinear rays with some common end points. The rays are called arms of the angle and the common end point is known as vertex of the angle.

(ii) Convert  $\frac{3\pi}{4}$  to degrees.

$$\begin{aligned}\text{Ans} \quad \frac{3\pi}{4} &= \frac{3\pi}{4} \times 1 \text{ radian} \\ &= \frac{3\pi}{4} \times \frac{180^\circ}{\pi} \\ &= 135^\circ\end{aligned}$$



(iii) Define projection.

**Ans** The projection of a given point on a line is the foot of  $\perp$  drawn from the point on that line. However, the projection of given point P on a line AB is the point P itself.

(iv) Define circle.

**Ans** A circle is the locus of a moving point P in a plane which is always equidistant from some fixed point O.

(v) Define secant.

**Ans** A secant is a straight line which cuts the circumference of a circle in two distinct points.

(vi) Define circumference of a circle.

**Ans** The boundary of a circle is called circumference.  $2\pi r$  is the circumference of a circle with radius r.

(vii) Define sector of a circle.

**Ans** The sector of a circle is an area bounded by any two radii and the arc intercepted between them.

(viii) Define radius of a circle.

**Ans** The distance from the centre of the circle to any point on the circle is called radius of the circle.

(ix) Define circum circle.

**Ans** The circle passing through the vertices of triangle ABC is known as circum circle, its radius as circum radius and centre as circum centre.

### (Part-II)

**NOTE:** Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

**Q.5.(a)** Solve the equation by completing square: (4)

$$7x^2 + 2x - 1 = 0$$

**Ans** For Answer see Paper 2015 (Group-I), Q.5(a).

**(b)** For what value of k, the expression  $k^2x^2 + 2(k+1)x + 4$  is perfect square. (4)

**Ans** Given,  $k^2x^2 + 2(k+1)x + 4$  (i)



Here,  $a = k^2$ ,  $b = 2(k + 1)$ ,  $c = 4$

$$\begin{aligned}\text{Discriminant} &= b^2 - 4ac \\ &= \{2(k + 1)\}^2 - 4(k^2)(4) \\ &= 4(k^2 + 1 + 2k) - 16k^2 \\ &= 4k^2 + 4 + 8k - 16k^2 \\ &= -12k^2 + 8k + 4\end{aligned}$$

As expression (i) is a perfect square (given), so the roots must be rational and equal. Thus,

$$\begin{aligned}\text{Discriminant} &= 0 \\ -12k^2 + 8k + 4 &= 0 \\ 12k^2 - 8k - 4 &= 0 \\ 12k^2 - 12k + 4k - 4 &= 0 \\ 12k(k - 1) + 4(k - 1) &= 0 \\ (k - 1)(12k + 4) &= 0\end{aligned}$$

$$k - 1 = 0$$

$$k = 1$$

$$12k + 4 = 0$$

$$12k = -4$$

$$k = \frac{-4}{12}$$

$$k = \frac{-1}{3}$$

Q.6.(a) If  $a : b = c : d$  ( $a, b, c, d \neq 0$ ) by using k-method,

show that  $\frac{a}{b} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}}$ . (4)

**Ans** Given,  $\frac{a}{b} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}}$  (I)

And given ratio,

$$a : b = c : d$$

$$\frac{a}{b} = \frac{c}{d}$$

By letting,  $\frac{a}{b} = k$  ;  $\frac{c}{d} = k$

$$a = bk ; c = dk$$



$$\text{L.H.S of (I)} = \frac{a}{b}$$

$$= \frac{bk}{b}$$

$$= k$$

$$\therefore a = bk$$

(II)

$$\text{R.H.S of (I)} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}}$$

$$= \sqrt{\frac{(bk)^2 + (dk)^2}{b^2 + d^2}}$$

$$\therefore a = bk \text{ and } c = dk$$

$$= \sqrt{\frac{b^2k^2 + d^2k^2}{b^2 + d^2}}$$

$$= \sqrt{\frac{k^2(b^2 + d^2)}{b^2 + d^2}}$$

$$= \sqrt{k^2}$$

$$= k$$

(III)

From II and III, we get

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{Hence, } \frac{a}{b} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}} \quad \text{Proved}$$

(b) Resolve into partial fraction:  $\frac{9}{(x-1)(x+2)^2}$  (4)

**Ans**  $\frac{9}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$

By multiplying both sides with  $(x-1)(x+2)^2$ , we get

$$\frac{9}{(x-1)(x+2)^2} (x-1)(x+2)^2 = \frac{A}{(x-1)} (x-1)(x+2)^2 + \frac{B}{(x+2)} (x-1)(x+2)^2 + \frac{C}{(x+2)^2} (x-1)(x+2)^2$$

$$9 = A(x+2)^2 + B(x-1)(x+2) + C(x-1) \quad \text{(I)}$$

To find the value of 'A'; Put  $x-1=0$  in (I)

$$x-1=0$$

$$x=1$$

$$9 = A(1+2)^2 + B(1-1)(1+2) + C(1-1)$$



$$9 = A(3)^2 + B(0)(3) + C(0)$$

$$9 = A(9) + 0 + 0$$

$$\frac{9}{9} = A$$

$$\Rightarrow A = 1$$

To find the value of 'c', put  $(x + 2)^2 = 0$  in (I):

$$(x + 2)^2 = 0$$

$$x + 2 = 0$$

$$x = -2$$

$$9 = A(-2 + 2)^2 + B(-2 - 1)(-2 + 2) + C(-2 - 1)$$

$$9 = A(3)^2 + B(0)(3) + C(0)$$

$$9 = A(0)^2 + B(-3)(0) + C(-3)$$

$$9 = 0 + 0 - 3C$$

$$\frac{9}{-3} = C$$

$$\Rightarrow \boxed{C = -3}$$

To find the value of 'B'

$$9 = A(x + 2)^2 + B(x - 1)(x + 2) + C(x - 1)$$

$$9 = A(x^2 + 4 + 4x) + B[x^2 + 2x - x - 2] + C(x - 1)$$

$$9 = Ax^2 + 4A + 4Ax + Bx^2 + Bx - 2B + Cx - C$$

$$9 = Ax^2 + Bx^2 + 4Ax + Bx + Cx + 4A - 2B - C \quad (\text{II})$$

By equating coefficients of  $x^2$  on both sides, we get

$$0 = A + B$$

$$0 = 1 + B$$

$$-1 = B$$

$$\Rightarrow \boxed{B = -1}$$

By putting the values of A, B and C in their relevant places, it is resolved that

$$\frac{9}{(x - 1)(x + 2)^2} = \frac{1}{x - 1} - \frac{1}{x + 2} - \frac{3}{(x + 2)^2}$$

**Q.7.(a)** If  $U = \{1, 2, 3, 4, \dots, 10\}$ ,  $A = \{1, 3, 5, 7, 9\}$ ,  
 $B = \{2, 3, 4, 5, 8\}$ , then prove that  $(B - A)' = B' \cup A$  (4)

**Ans**  $\Rightarrow$  L.H.S =  $(B - A)'$



$$\text{Firstly, } B - A = \{2, 3, 4, 5, 8\} - \{1, 3, 5, 7, 9\} \\ = \{2, 4, 8\}$$

$$(B - A)' = U - (B - A) = \{1, 2, 3, 4, \dots, 10\} - \{2, 4, 8\} \\ = \{1, 3, 5, 6, 7, 9, 10\}$$

$$\text{R.H.S} = B' \cup A$$

$$\text{Firstly, } B' = U - B = \{1, 2, 3, 4, \dots, 10\} - \{2, 3, 4, 5, 8\} \\ = \{1, 6, 7, 9, 10\}$$

$$B' \cup A = \{1, 6, 7, 9, 10\} \cup \{1, 3, 5, 7, 9\} \\ = \{1, 3, 5, 6, 7, 9, 10\}$$

$$\text{So, L.H.S} = \text{R.H.S}$$

(b) Find standard deviation 'S':

(4)

9, 3, 8, 8, 9, 8, 9, 18

Ans

$$n = 8$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{9 + 3 + 8 + 8 + 9 + 8 + 9 + 18}{8}$$

$$\bar{x} = \frac{72}{8}$$

$$\bar{x} = 9$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
9	0	0
3	-6	36
8	-1	1
8	-1	1
9	0	0
8	-1	1
9	0	0
18	9	81
SUM	0	120

Standard Deviation:

$$S = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$



$$= \sqrt{\frac{120}{8}}$$

$$= \sqrt{15}$$

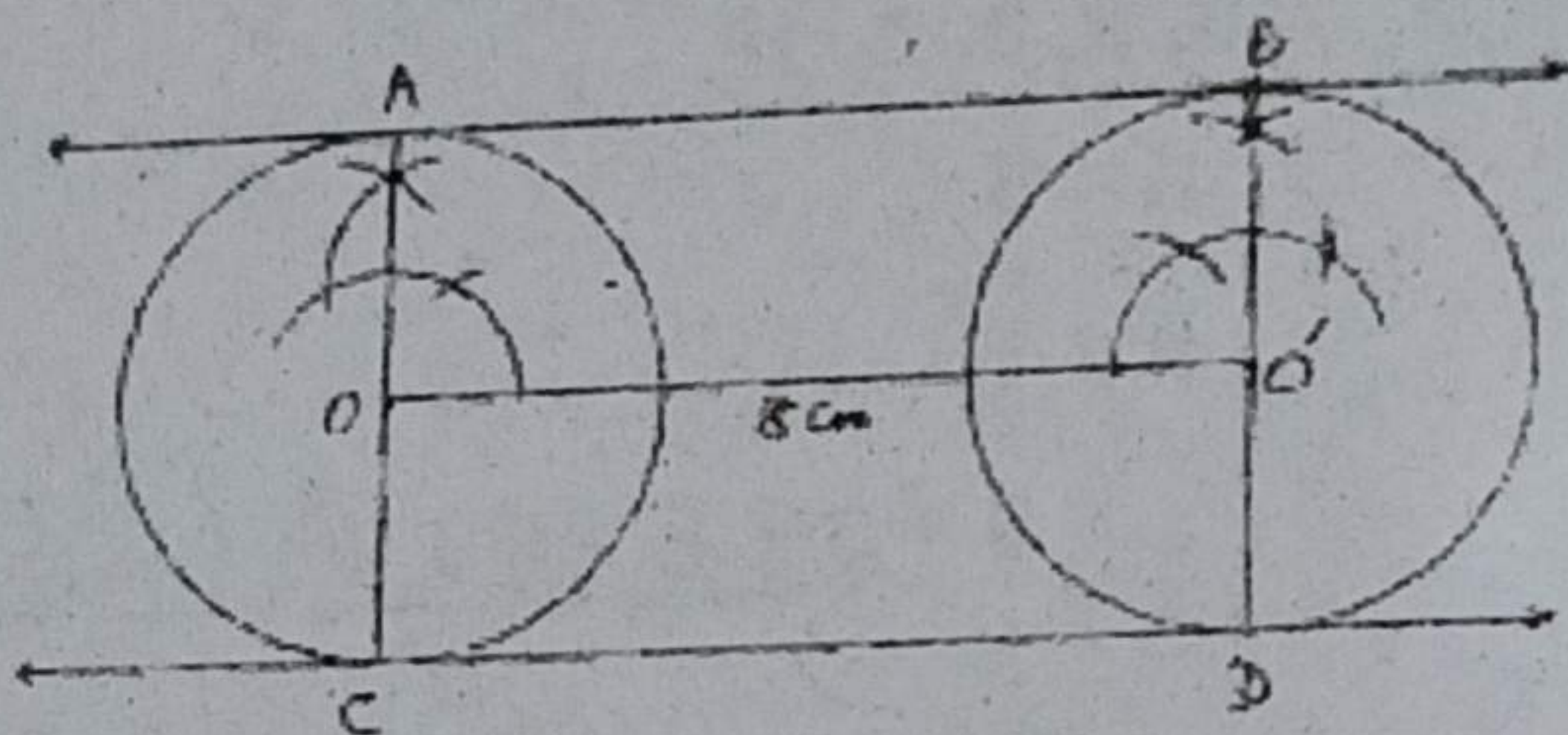
$$S = 3.87$$

Q.8.(a) Prove that:  $\frac{1 + \sin \theta}{1 - \sin \theta} - \frac{1 - \sin \theta}{1 + \sin \theta} = 4 \tan \theta \sec \theta$ .

**Ans** For Answer see Paper 2017 (Group-II), Q.8.(a).

(b) Two equal circles are at 8 cm apart. Draw two direct common tangents of this pair of circles. (4)

**Ans**



### Step of Construction:

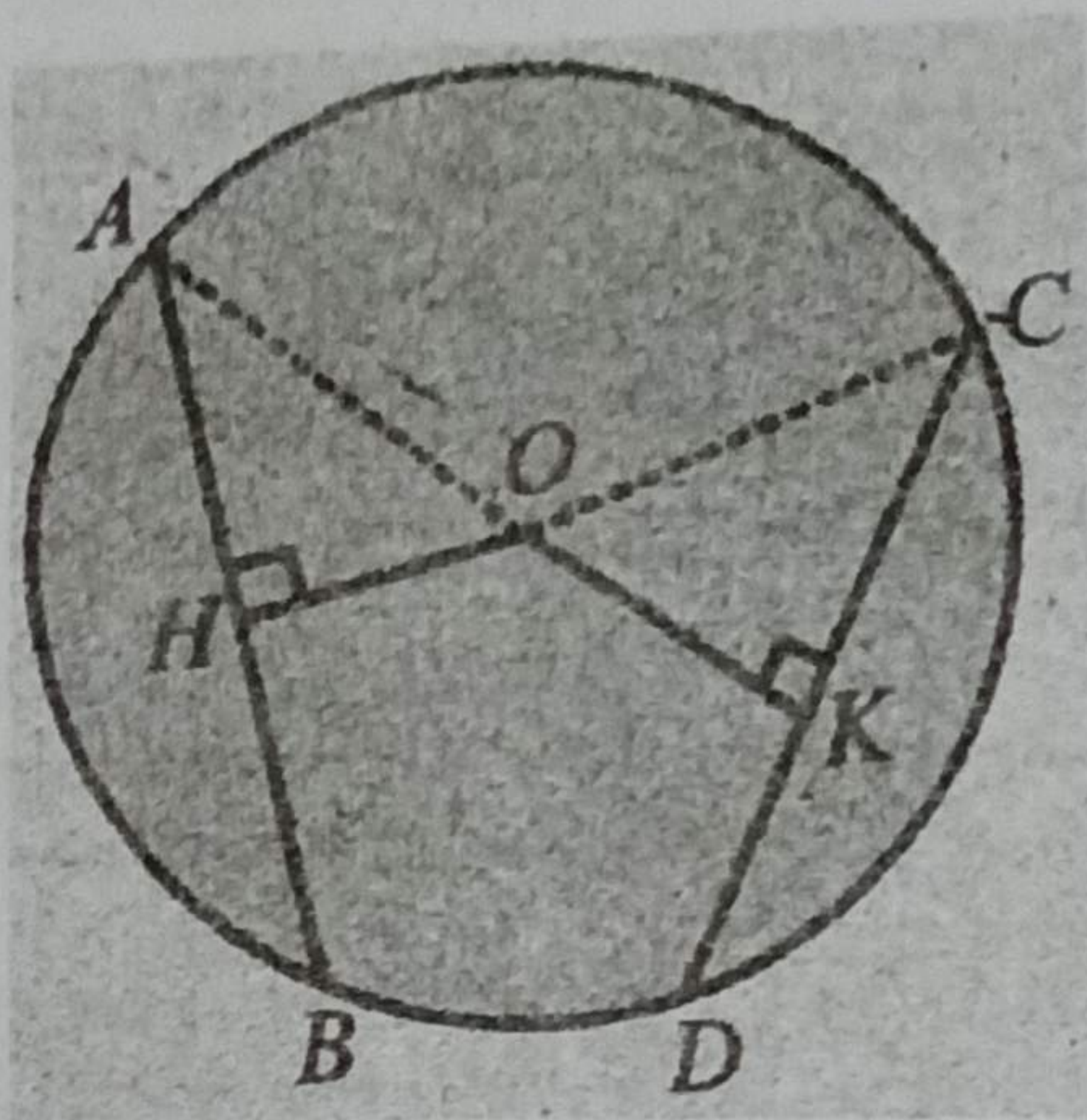
- (i) Draw a line segment of 8 cm length.
- (ii) Draw two circles of equal size on their centres O and O'.
- (iii) Take  $\overline{OA} \perp \overline{OO'}$  and produce it towards O. Then,  $\overline{OA}$  meets the circle at C.
- (iv) Take  $\overline{O'B} \perp \overline{OO'}$  and produce it towards O'.  $\overline{O'B}$  meets the circle at D.
- (v) Join A with B and C with D, and produce these both sides.

Thus, AB and CD are the required common external tangents.



Q.9. Prove that two chords of a circle which are equidistant from the centre, are congruent.

**Ans**



**Given:**

$\overline{AB}$  and  $\overline{CD}$  are two equal chords of a circle with centre at O.

So that  $\overline{OH} \perp \overline{AB}$  and  $\overline{OK} \perp \overline{CD}$ .

**To prove:**

$$m\overline{OH} = m\overline{OK}$$

**Construction:**

Join O with A and O with C.

So that we have  $\angle$ rt $\Delta^s$  OAH and OCK.

**Proof:**

Statements	Reasons
$\overline{OH}$ bisects chord $\overline{AB}$ i.e., $m\overline{AH} = \frac{1}{2} m\overline{AB}$ (i)	$\overline{OH} \perp \overline{AB}$ By Theorem 3
Similarly $\overline{OK}$ bisects chord $\overline{CD}$ i.e., $m\overline{CK} = \frac{1}{2} m\overline{CD}$ (ii)	$\overline{OK} \perp \overline{CD}$ By Theorem 3
But $m\overline{AB} = m\overline{CD}$ (iii)	Given



Hence  $m\overline{AH} = m\overline{CK}$  (iv)

Now in  $\angle rt \Delta^s OAH \leftrightarrow OCK$

hyp  $\overline{OA} = \text{hyp } \overline{OC}$

$m\overline{AH} = \overline{CK}$

$\therefore \Delta OAH \cong \Delta OCK$

$\Rightarrow m\overline{OH} = m\overline{OK}$

Using (i), (ii) & (iii)

Given  $\overline{OH} \perp \overline{AB}$  and  $\overline{OK} \perp \overline{CD}$

Radii of the same circle

Already proved in (iv)

H.S postulate

OR

Prove that the opposite angles of any quadrilateral inscribed in a circle are supplementary.

**Ans** For Answer see Paper 2016 (Group-I), Q.9(OR).



$$\begin{aligned}y(y-2) - 2(y-2) &= 0 \\(y-2)(y-2) &= 0 \\y-2 &= 0 & y-2 &= 0 \\ \boxed{y=2} & & \boxed{y=2}\end{aligned}$$

So finally  $\boxed{y=2}$

Put the value of 'y' in equation (i),

$$\begin{aligned}\frac{x}{x-3} &= y \\ \frac{x}{x-3} &= 2 \\ x &= 2(x-3) \\ x &= 2x-6 \\ 2x-6-x &= 0 \\ x-6 &= 0 \\ \boxed{x=6}\end{aligned}$$

Therefore, solution set is  $\{6\}$ .

(b) If  $\alpha$  and  $\beta$  are the roots of the equation  $lx^2 + mx + n = 0$ , find the value of  $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ . (4)

**Ans** Given

$$lx^2 + mx + n = 0$$

Here,  $a = l$ ,  $b = m$ ,  $c = n$

$$\text{Then, } \alpha + \beta = \frac{-b}{a} = \frac{-m}{l}$$

$$\alpha\beta = \frac{c}{a} = \frac{n}{l}$$

So,

$$\begin{aligned}\frac{1}{\alpha^2} + \frac{1}{\beta^2} &= \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} \\ &= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}\end{aligned}$$