10th Class 2018

Math (Science)	Group-II	PAPER-II
Time: 20 Minutes	(Objective Type)	Max. Marks: 15

Note: Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1- If $u \propto v^2$ then:

(a) $u = v^2$

(b) $u = k v^2$ ✓

(c) $u v^2 = k$

(d) $u v^2 = 1$

2- A set with no element is called:

(a) Subset

(b) Empty set ✓

(c) Singleton set

(d) Super set

3- The number of terms in a standard quadratic equation $ax^2 + bx + c = 0$ is:

(a) 1

(b) 2

(c) 3 ✓

(d) 4

4- A frequency polygon is a many sided _____ :

(a) Closed figure ✓

(b) Rectangle

(c) Circle

(d) Triangle

5- $\sec \theta \cot \theta =$ _____ :

(a) $\sin \theta$

(b) $\dfrac{1}{\cos \theta}$

(c) $\dfrac{1}{\sin \theta}$ ✓

(d) $\dfrac{\sin \theta}{\cos \theta}$

6- The arcs opposite to incongruent central angles of a circle are always _____ :

(a) Parallel

(b) Perpendicular

(c) Congruent

(d) Incongruent ✓

7- The symbol for a triangle is denoted by:

(a) $\angle$

(b) $\triangle$ ✓

(c) $\perp$

(d) $\odot$

8- The length of the diameter of a circle is how many times the radius of the circle:

(a) 4 times

(b) 3 times

(c) 2 times √

(d) 1 time

9- If $\frac{a}{b} = \frac{c}{d}$, then componendo property is:

(a) $\frac{a}{a+b} = \frac{c}{c+d}$ √

(b) $\frac{a}{a-b} = \frac{c}{c-d}$

(c) $\frac{ad}{bc}$

(d) $\frac{a-b}{b} = \frac{c-d}{d}$

10- A circle has only one _____ :

(a) Secant

(b) Chord

(c) Diameter

(d) Centre √

11- How many common tangents can be drawn for two touching circles:

(a) 1

(b) 2

(c) 3 √

(d) 4

12- If $A \subseteq B$ then $A \cap B$ is equal to:

(a) A √

(b) B

(c) ϕ

(d) $A \cup B$

13- The discriminant of $ax^2 + bx + c = 0$ is:

(a) $b^2 - 4ac$ √

(b) $b^2 + 4ac$

(c) $-b^2 + 4ac$

(d) $-b^2 - 4ac$

14- A fraction in which the degree of numerator is less than the degree of the denominator is called:

(a) An equation

(b) An improper fraction

(c) An identity

(d) A proper fraction √

15- Product of cube roots of unity is:

(a) 0

(b) 1 √

(c) −1

(d) 3

10th Class 2018

Math (Science)	Group-II	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: 12

(i) Write the quadratic equation $\frac{x}{x+1} + \frac{x+1}{x} = 6$.

Ans Given,

$$\frac{x}{x+1} + \frac{x+1}{x} = 6$$

$$\frac{x(x) + (x+1)(x+1)}{(x+1)(x)} = 6$$

$$\frac{x^2 + (x+1)^2}{x^2 + x} = 6$$

$$x^2 + x^2 + 1 + 2x = 6(x^2 + x)$$

$$2x^2 + 2x + 1 = 6x^2 + 6x$$

$$0 = 6x^2 + 6x - 2x^2 - 2x - 1$$

$$0 = 4x^2 + 4x - 1$$

$$\Rightarrow \qquad \boxed{4x^2 + 4x - 1 = 0} \quad \text{Ans}$$

(ii) Write the standard quadratic equation and also write quadratic formula to solve it.

Ans The standard quadratic equation is:

$$ax^2 + bx + c = 0$$

Quadratic formula to solve it

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(iii) Find the sum and product of the roots of the equation $2px^2 + 3qx - 4r = 0$ without solving.

Ans Given, $2px^2 + 3qx - 4r = 0$

Here, $a = 2p$, $b = 3q$, $c = -4r$

$$\text{Sum of the roots} = \frac{-b}{a}$$

$$= \frac{-3q}{2p}$$

$$\text{Product of the roots} = \frac{c}{a}$$

$$= \frac{-4r}{2p}$$

$$= \frac{-2r}{p}$$

(iv) Form a quadratic equation whose roots are $3 + \sqrt{2}$ and $3 - \sqrt{2}$.

Ans Here, $\alpha = 3 + \sqrt{2}$

$$\beta = 3 - \sqrt{2}$$

$$\begin{aligned}
\text{Sum of the roots} &= \alpha + \beta \\
&= (3 + \sqrt{2}) + (3 - \sqrt{2}) \\
&= 3 + \sqrt{2} + 3 - \sqrt{2} \\
&= 6
\end{aligned}$$

$$\begin{aligned}
\text{Product of the roots} &= \alpha\beta \\
&= (3 + \sqrt{2})(3 - \sqrt{2}) \\
&= (3)^2 - (\sqrt{2})^2 \\
&= 9 - 2 \\
&= 7
\end{aligned}$$

To find quadratic equation,

$$x^2 - Sx + P = 0$$

By putting the values, we get the required quadratic equations:

$$x^2 - 6x + 7 = 0$$

(v) Evaluate: $(1 - 3\omega - 2\omega^2)^5$

Ans

$$\begin{aligned}
(1 - 3\omega - 3\omega^2)^5 &= [1 - 3(\omega + \omega^2)]^5 \\
&= [1 - 3(-1)]^5 \qquad \because \ \omega + \omega^2 = -1 \\
&= (1 + 3)^5 \\
&= 4^5 \\
&= 1024
\end{aligned}$$

(vi)   Define synthetic division.

Ans Synthetic division is the process of finding the quotient and remainder, when a polynomial is divided by a linear polynomial. In fact, synthetic division is simply a short-cut of long division method.

(vii)   Find p, if 12, p and 3 are in continued proportion.

Ans Since 12, p and 3 are in continued proportion.

$$\therefore \quad 12 : p :: p : 3$$
$$i.e., \quad (p)(p) = (12)(3)$$
$$p^2 = 36$$

Thus, $p = \pm 6$

(viii)   Find the ratio x : y, if $3(4x - 5y) = 2x - 7y$.

Ans Given,

$$3(4x - 5y) = 2x - 7y$$
$$12x - 15y = 2x - 7y$$
$$12x - 2x = -7y + 15y$$
$$10x = 8y$$
$$\Rightarrow \qquad \frac{x}{y} = \frac{8}{10}$$
$$\frac{x}{y} = \frac{4}{5}$$
$$x : y = 4 : 5 \quad \text{Ans.}$$

(ix)   Find a fourth proportional to 5, 8, 15.

Ans Let a fourth proportional is x;

So,

$$5 : 8 :: 15 : x$$

Product of Extremes = Product of Means

$$5(x) = 8 \times 15$$
$$x = \frac{8 \times 15}{5}$$
$$x = 8 \times 3$$
$$x = 24$$

So, the fourth proportional is $x = 24$.

3. Write short answers to any SIX (6) questions: 12

(i) What is an improper fraction?

Ans A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called an improper fraction if degree of the polynomial $N(x)$ is greater or equal to the degree of the polynomial $D(x)$.

For example:
$$\frac{5x}{x+2}, \frac{3x^2+2}{x^2+7x+12}, \frac{6x^4}{x^3+1}.$$

(ii) Find partial fraction of $\frac{3}{(x+1)(x-1)}$.

Ans Let,
$$\frac{3}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} \qquad \text{(i)}$$

By multiplying $(x+1)(x-1)$, we get
$$\frac{3}{(x+1)(x-1)}(x+1)(x-1) = \frac{A}{(x+1)}(x+1)(x-1) +$$
$$\frac{B}{(x-1)}(x+1)(x-1)$$

$$3 = A(x-1) + B(x+1) \qquad \text{(ii)}$$

As, $\quad x - 1 = 0$
$$x = 1$$

Put $x = 1$ in (ii),
$$3 = A(1-1) + B(1+1)$$
$$3 = A(0) + 2B$$
$$3 = 2B$$
$$\frac{3}{2} = B$$

$\Rightarrow \qquad B = \frac{3}{2}$

And $\quad x + 1 = 0$
$$x = -1$$

Put $x = -1$ in (ii),
$$3 = A(-1-1) + B(-1+1)$$

$$3 = A(-2) + B(0)$$
$$3 = -2A$$
$$\frac{3}{-2} = A$$
$$\Rightarrow \quad A = \frac{-3}{2}$$

By putting the values of A and B in (i),

$$\frac{3}{(x + 1)(x - 1)} = \frac{-3}{2(x + 1)} + \frac{3}{2(x - 1)}$$

(iii)   If X = {1, 4, 7, 9} and Y = {2, 4, 5, 9} then find Y ∩ X.

Ans Given, X = {1, 4, 7, 9} , Y = {2, 4, 5, 9}

$$Y \cap X = \{2, 4, 5, 9\} \cap \{1, 4, 7, 9\}$$
$$= \{4, 9\}$$

(iv)   If X = {1, 3, 5, 7, -----, 9} Y = {0, 2, 4, 6, 8, -----, 20} and
Z = {2, 3, 5, 7, 11, 13, 17, 19, 23} then find (X∩Y)∩Z.

Ans $(X \cap Y) = (\{1, 3, 5, 7, ..., 9\} \cap \{0, 2, 4, 6, 8, ..., 20\}) \cap Z$
$$= \{\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$
$$= \{\}$$

(v)   Find a and b if (2a + 5, 3) = (7, b - 4).

Ans Given (2a + 5, 3) = (7, b - 4)

By comparing, we get

Firstly,

$$2a + 5 = 7$$
$$2a = 7 - 5$$
$$2a = 2$$
$$a = \frac{2}{2}$$

$$\boxed{a = 1}$$

And
$$3 = b - 4$$
$$3 + 4 = b$$
$$7 = b$$
$$\Rightarrow \quad \boxed{b = 7}$$

(vi) **Define an onto function.**

Ans A function $f : A \rightarrow B$ is called an onto function, if every element of set B is an image of at least one element of set A *i.e.*, Range of $f = B$.

(vii) **Define a frequency distribution.**

Ans A frequency distribution is a tabular arrangement for classifying data into different groups and the number of observations falling in each group corresponds to the respective group.

(viii) **Find arithmetic mean by direct method:**
$$200, 225, 350, 375, 270, 320, 290.$$

Ans The arithmetic Mean:

$$\bar{X} = \frac{\Sigma X}{n}$$

$$= \frac{200 + 225 + 350 + 375 + 270 + 320 + 290}{7}$$

$$= \frac{2030}{7}$$

$$\boxed{\bar{X} = 290}$$

(ix) **For the following data, find the harmonic mean:**

x	12	5	8	4

Ans

X	$\frac{1}{X}$
12	0.0833
5	0.2
8	0.125
4	0.25
	0.6583

$$\text{H.M} = \frac{n}{\Sigma\left(\frac{1}{x}\right)} = \frac{4}{0.6583}$$

$$H.M = 6.0763$$

4. **Write short answers to any SIX (6) questions: 12**

(i) Verify the identity: $(1 - \sin\theta)(1 + \sin\theta) = \cos^2\theta$

Ans Given,

$$(1 - \sin\theta)(1 + \sin\theta) = \cos^2\theta$$
$$\begin{aligned}
\text{L.H.S} &= (1 - \sin\theta)(1 + \sin\theta) \\
&= (1)^2 - (\sin\theta)^2 \\
&= 1 - \sin^2\theta \\
&= (\sin^2\theta + \cos^2\theta) - \sin^2\theta \\
&= \sin^2\theta + \cos^2\theta - \sin^2\theta \\
&= \cos^2\theta \\
&= \text{R.H.S}
\end{aligned}$$

(ii) How many minutes are there in two right angles?

Ans As we know that:

1 degree = 60 minutes

Two right angles have 180 degrees

Thus

Two right angles = 180×60 minutes

$= 10,800$ minutes

(iii) Find 'r', when $l = 52$ cm, $\theta = 45°$.

Ans As we know that,

$$\theta = 45 \times \frac{\pi}{180} = \frac{\pi}{4}$$
$$\begin{aligned}
r &= \frac{l}{\theta} \\
&= l \div \theta \\
&= 52 \div \frac{\pi}{4} \\
&= 52 \times \frac{4}{\pi} \\
&= 52 \times 1.273 \\
\boxed{r &= 66.21}
\end{aligned}$$

(iv) **What is meant by zero dimension?**

Ans The projection of a finite line on another line is the portion of the latter intercepted between the projection of ends of the given finite line. However, projection of a vertical line on another line is the join of these two interesting lines, which is of zero dimension.

(v) **Define circumference.**

Ans The length of the boundary of the circle is called the circumference.

(vi) **Define secant.**

Ans A secant is a straight line which cuts the circumference of a circle in two distinct points.

(vii) **Define chord of a circle.**

Ans The join of any two points on the circumference of the circle is called its chord.

(viii) **Define cyclic quadrilateral.**

Ans A quadrilateral is called cyclic when a circle can be drawn through its four vertices.

(ix) **Define an arc.**

Ans A part of circumference of a circle is called an arc.

(Part-II)

NOTE: Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve the equation: (4)

$$5x^{1/2} = 7x^{1/4} - 2$$

Ans Let
$$x^{1/4} = y$$
$$(x^{1/4})^2 = (y)^2$$
$$x^{1/2} = y^2$$

By putting the values in the given expression, we get
$$5(y^2) = 7(y) - 2$$
$$5y^2 = 7y - 2$$
$$5y^2 - 7y + 2 = 0$$
$$5y^2 - 5y - 2y + 2 = 0$$
$$5y(y - 1) - 2(y - 1) = 0$$

$$(5y - 2)(y - 1) = 0$$

$$5y - 2 = 0 \quad ; \quad y - 1 = 0$$

$$5y = 2 \quad ; \quad \boxed{y = 1}$$

$$\boxed{y = \frac{2}{5}}$$

As $\quad y = x^{1/4}$

So, $\quad x^{1/4} = 1 \qquad ; \qquad x^{1/4} = \frac{2}{5}$

Taking square on   ;   Taking square on
both sides   both sides

$$(x^{1/4})^2 = (1)^2 \qquad ; \qquad (x^{1/4})^2 = \left(\frac{2}{5}\right)^2$$

$$x^{1/2} = 1 \qquad ; \qquad x^{1/2} = \frac{4}{25}$$

Again taking square   ;   Again taking square
on both sides   on both sides

$$(x^{1/2})^2 = (1)^2 \qquad ; \qquad (x^{1/2})^2 = \left(\frac{4}{25}\right)^2$$

$$x = 1 \qquad ; \qquad x = \frac{16}{625}$$

$$S.S = \left\{1, \frac{16}{625}\right\}$$

(b) Find the value of h using synthetic division, if 3 is the zero of the polynomial $2x^3 - 3hx^2 + 9$. **(4)**

Ans As 3 is the zero of the polynomial $2x^3 - 3hx^2 + 9$.

$$P(x) = 2x^3 - 3hx^2 + 9$$
$$= 2x^3 - 3hx^2 + 0x + 9$$

And $\quad a = 3$

So,

3	2	−3h	0	9
	↓	6	−9h + 18	−27h + 54
	2	−3h + 6	−9h + 18	−27h + 63

As 3 is the zero of polynomial, then R = 0

(b) Resolve into partial fractions: **(4)**

$$\frac{1}{(x^2-1)(x+1)}$$

Ans $\frac{1}{(x^2-1)(x+1)} = \frac{1}{(x-1)(x+1)(x+1)}$

$$= \frac{1}{(x+1)^2(x-1)}$$

So,

$$\frac{1}{(x+1)^2(x-1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1} \qquad \text{(i)}$$

By multiplying $(x+1)^2(x-1)$, we get

$$1 = A(x+1)(x-1) + B(x-1) + C(x+1)^2 \qquad \text{(ii)}$$

$$1 = A(x^2-1) + B(x-1) + C(x^2+2x+1) \qquad \text{(iii)}$$

Put $x+1 = 0$, i.e., $x = -1$ in (ii),

$$1 = A((-1)^2-1) + B(-1-1) + C((-1)^2+2(-1)+1)$$

$$1 = A(0) + B(-1-1) + C(0)$$

$$1 = B(-1-1)$$

$$1 = -2B$$

$$\boxed{B = \frac{-1}{2}}$$

Put $x-1 = 0$, i.e., $x = 1$ in (ii),

$$1 = C(1+1)^2$$

$$1 = C(2)^2$$

$$1 = 4C$$

$$\Rightarrow \quad \boxed{C = \frac{1}{4}}$$

By comparison the coefficients of x^2 in (iii),

$$0 = A + C$$

Put the value of C,

$$0 = A + \frac{1}{4}$$

$$\Rightarrow \quad \boxed{A = \frac{-1}{4}}$$

Put the values of A, B, C in (i),

$$R = -27h + 63 = 0$$
$$-27h = -63$$
$$h = \frac{-63}{-27}$$
$$h = \frac{7}{3}$$

Q.6.(a) Using componendo-dividendo theorem, solve the equation $\dfrac{\sqrt{x+3} + \sqrt{x-3}}{\sqrt{x+3} - \sqrt{x-3}} = \dfrac{4}{3}$. **(4)**

Ans Given equation is $\dfrac{\sqrt{x+3} + \sqrt{x-3}}{\sqrt{x+3} - \sqrt{x-3}} = \dfrac{4}{3}$

By using componendo-dividendo theorem,

$$\frac{\sqrt{x+3} + \sqrt{x-3} + \sqrt{x+3} - \sqrt{x-3}}{\sqrt{x+3} + \sqrt{x-3} - \sqrt{x+3} + \sqrt{x-3}} = \frac{4+3}{4-3}$$

$$\frac{2\sqrt{x+3}}{2\sqrt{x-3}} = \frac{7}{1}$$

$$\sqrt{\frac{x+3}{x-3}} = 7$$

Squaring both sides, we get

$$\frac{x+3}{x-3} = 49$$

$$x + 3 = 49(x - 3)$$
$$x + 3 = 49x - 147$$
$$x - 49x = -147 - 3$$
$$-48x = -150$$
$$48x = 150$$
$$x = \frac{150}{48}$$
$$x = \frac{25}{8}$$

$$\frac{1}{(x+1)^2(x-1)} = \frac{-1}{4(x+1)} - \frac{1}{2(x+1)^2} + \frac{1}{4(x-1)}$$

So, $$\frac{1}{(x+1)^2(x-1)} = \frac{1}{4(x-1)} - \frac{1}{4(x+1)} - \frac{1}{2(x+1)^2}$$

Q.7.(a) If U = {1, 2, 3, 4, - - - -, 10}, A = {1, 3, 5, 7, 9} and B = {1, 4, 7, 10} then verify that B − A = B ∩ A'. (4)

Ans
$$
\begin{aligned}
\text{L.H.S} &= B - A \\
&= \{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\} \\
&= \{4, 10\}
\end{aligned}
$$

R.H.S = B ∩ A'

Now A' = U − A

$$
\begin{aligned}
&= \{1, 2, 3, 4, ..., 10\} - \{1, 3, 5, 7, 9\} \\
&= \{2, 4, 6, 8, 10\}
\end{aligned}
$$

$$
\begin{aligned}
\text{R.H.S} &= B \cap A' \\
&= \{1, 4, 7, 10)\} \cap \{2, 4, 6, 8, 10\} \\
&= \{4, 10\}
\end{aligned}
$$

So, L.H.S = R.H.S

(b) Calculate variance for the data: (4)

10, 8, 9, 7, 5, 12, 8, 6, 8, 2

Ans

$$\bar{X} = \frac{\Sigma X}{n}$$

$$= \frac{10 + 8 + 9 + 7 + 5 + 12 + 8 + 6 + 8 + 2}{10}$$

$$= \frac{75}{10}$$

$$\boxed{\bar{X} = 7.5}$$

X	$x - \bar{X}$	$(x - \bar{X})^2$
10	2.5	6.25
8	0.5	0.25
9	1.5	2.25
7	−0.5	0.25
5	−2.5	6.25
12	4.5	20.25

8	0.5	0.25
6	−1.5	2.25
8	0.5	0.25
2	−5.5	30.25
		68.5

$$\text{Variance } (x) = \frac{\Sigma(x - \bar{X})^2}{n}$$

$$= \frac{68.5}{10}$$

$$\boxed{\text{Var } (x) = 6.85}$$

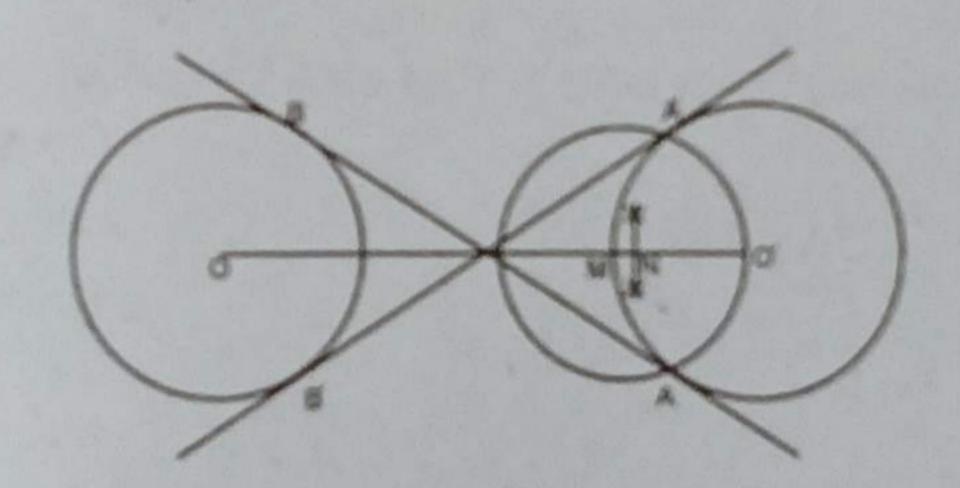**Q.8.(a) Verify:** $(\tan\theta + \cot\theta)\tan\theta = \sec^2\theta.$ (4)

Ans

$$\text{L.H.S} = (\tan\theta + \cot\theta)\tan\theta$$
$$= \tan\theta\tan\theta + \tan\theta\cot\theta$$
$$= \tan^2\theta + \frac{\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta}{\cos^2\theta} + 1$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}$$
$$= \frac{1}{\cos^2\theta}$$
$$= \sec^2\theta$$
$$= \text{R.H.S}$$

(b) Draw two equal circles of each radius 2.4 cm. If the distance between their centres is 6 cm, then draw their transverse tangents? (4)

Ans

Steps of Construction:
1. Draw a line segment $mOO' = 6$ cm.
2. Draw two circles of 2.4 cm radius on O and O'.
3. Take M as the mid-point of $\overline{OO'}$ and N as the mid-point of $\overline{MO'}$.
4. Draw a circle with centre at N and a radius $\overline{NO'}$. This circle intersects the circle AA'.
5. Join A' with M and produce towards M, it touch the second circle at B'.
6. Join A with M and produce towards M. AM produced touches the second circle at B.

   So, A'B' are the required tangents.

Q.9. Prove that a straight line, drawn from the centre of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord. **(8)**

Ans For Answer see Paper 2015 (Group-II), Q.9.

OR

Prove that the measure of a central angle of a minor arc of a circle is double that of the angle subtended by the corresponding major arc.

Ans For Answer see Paper 2017 (Group-I), Q.9.(OR).