MATHEMATICS HSSC-II (2015)

SECTION-A (Marks 20)

Time allowed: 25 Minutes

NOTE: - Section-A is compulsory. All parts of this section are to be answered on the question paper itself. It should be completed in the first 25 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q1. Circle the correct option i.e. A / B / C / D. Each part carries one mark.

i. **What is the range of f^{-1}, when $f(x) = 2 + \sqrt{x-1}$?**

A. $[1, \infty)$
B. $(-\infty, -1]$
C. $[-1, 1]$
D. $[2, \infty)$

ii. $\lim_{x \to 0} \frac{2-3x}{\sqrt{3+4x^2}} = ?$

A. $\frac{3}{2}$ B. $\frac{-3}{2}$
C. $\pm\frac{3}{2}$ D. None of these

iii. $Ins\,(1+x) = ?$

A. $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$
B. $1 - \frac{x^2}{2!} + \frac{x^4}{4} - \frac{x^6}{6} + \cdots$
C. $x + \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$
D. $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

iv. **If P is the perimeter of square and A its area then A=?**

A. $\frac{P^2}{4}$ B. $\frac{P^2}{8}$
C. $\frac{P^2}{16}$ D. $16P^2$

v. $\int \frac{dx}{x^2-9} = ?$

A. $\frac{1}{6} ln\left(\frac{x+3}{x-3}\right) + C$ B. $\frac{1}{6} ln\left(\frac{3+x}{3-x}\right) + C$
C. $\frac{1}{6} tan^{-1}\frac{x}{3} + C$ D. $\frac{1}{3} tan^{-1}\frac{x}{3} + C$

vi. **If $f(x) = \frac{2x+1}{x-1}$ then $f^{-1}(x) = ?$**

A. $\frac{1}{2}\left(\frac{x+1}{x-1}\right)$ B. $\frac{1}{2}\left(\frac{x-1}{x+1}\right)$
C. $\frac{1}{2}\left(\frac{x-2}{x-2}\right)$ D. None of these

vii. **$x = asec\theta$, $y = btan\theta$ are the parametric equation of:**

A. Ellipse B. Circle
C. Hyperbola D. Parabola

viii. $\frac{d}{dx}(\sin^{-1}\frac{a}{x}) = ?$

A. $-\frac{1}{x^2} cos\frac{a}{x}$ B. $\frac{1}{x} cos\frac{a}{x}$
C. $\frac{1}{a} cos\frac{a}{x}$ D. None of these

ix. $\int_{-1}^{3}(x^3 + 3x^2)dx = ?$

A. 28 B. 48
C. 58 D. 20

x. **Distance of the point (x,y) from x-axis is:**

A. x B. y
C. $|x|$ D. $|y|$

xi. **The slope of the line $2x + 3y = 7$ is:**

A. $\frac{2}{3}$ B. $\frac{1}{3}$
C. $\frac{-2}{3}$ D. $\frac{-1}{2}$

xii. **The coordinates of the point that divides the join of A (-6, 3) and B (5, -2) in the ration 2:3,**

A. $\left(1, \frac{9}{5}\right)$ B. $\left(\frac{-8}{5}, 1\right)$
C. $\left(0, \frac{9}{5}\right)$ D. None of these

xiii. **The two lines ℓ_1 and ℓ_2 with respective slope m_1 and m_2 are perpendicular if:**

A. $m_1 + m_2 = 0$ B. $m_1 m_2 = 1$
C. $m_1 m_2 = -1$ D. $m_1 - m_2 = 0$

xiv. **A region which is restricted to the first quadrant is called:**

A. Maximum region
B. Minimum region
C. Feasible region
D. Objective function

xv. **The centre of a circle $x^2 + y^2 + 6x - 10y - 15 = 0$ is:**

A. (5 , 3) B. (5, -3)
C. (-3, 5) D. (3, 5)

xvi. **If e<1, the conic is called:**

A. Parabola B. Circle
C. Hyperbola D. Ellipse

xvii. **The focus of a parabola $x^2 = -16y$ is:**

A. $(-4, 0)$ B. $(0, -4)$
C. $(4, -4)$ D. $(0, \pm 4)$

xviii. **If A = $\underline{i} + \sqrt{3}\,\underline{j}$, then the unit vector $\hat{A}$ is:**

A. $\frac{-i+\sqrt{3}j}{2}$ B. $\frac{i+\sqrt{3}j}{2}$
C. $\frac{i-\sqrt{3}j}{2}$ D. None of these

xix. **A vector perpendicular to $2\underline{i} - \underline{j} + \underline{k}$ and $4\underline{i} + 2\underline{j} + 8\underline{k}$ is:**

A. $-\underline{i} + 6\underline{j} + 8\underline{k}$ B. $-10\underline{i} - 12\underline{j} + 8\underline{k}$
C. $\underline{i} + 6\underline{j} + 8\underline{k}$ D. $-\underline{i} + 6\underline{j} + 8\underline{k}$

xx. **$(2\underline{i}, 3\underline{i}) \times \underline{k}$ is:**

A. $2\underline{i} - \underline{j}$ B. $2\underline{i} - 3\underline{k}$
C. 0 D. None of these

MATHEMATICS HSSC-II (2015)

Time allowed: 2:35 Hours | Total Marks : 80

Note: Attempt any ten parts from Section 'B' and any five questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet-B if required. Write your answers neatly and legibly.

SECTION - B (Marks 40)

Q.2 Attempt any TEN parts. All parts carry equal marks.

(i) Evaluate $\lim_{x\to 0} \frac{\sin x^0}{x}$

(ii) Graph the curve that has the parametric equations given below:
$x = t - 1, \quad y = 2t - 1, \quad -1 \leq t \leq 5$
where "t" is a parameter.

(iii) Prove that $y\frac{dy}{dx} + x = 0$ If
$x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$

(iv) If $y = \tan(p\tan^{-1}x)$ show that:
$(1+x^2)y_1 - p(1+y^2) = 0$

(v) Find the point on the curve $y = x^2 + 1$ that is closest to the point $(18, 1)$.

(vi) Evaluate $\int \frac{x^2}{3+x^2}dx$?

(vii) Evaluate $\int \frac{(1-\sin x)}{(1-\cos x)}e^x dx$

(viii) Evaluate $\int_0^{\frac{\pi}{4}} \sec x(\sec x + \tan x)dx$.

(ix) Find an equation of the line through (5,-8) and perpendicular to the joint A (-15,-8) and B (10, 7).

(x) Find the point which is equidistant from the points A (5,3), B (-2,2) and C (4,2).

(xi) Find centre and radius of circle with the given equation
$4x^2 + 4y^2 - 8x + 12y - 25 = 0$

(xii) Find an equation of the ellipse with foci $(\pm 3, 0)$ and minor axis of length 10.

(xiii) A parabolic arch has a 100 m base and height 25m. Find the height of arch at the point 30 m from the centre of the base.

(xiv) Prove that the altitudes of a triangle are concurrent.

SECTION – C

Q. 3 If $f(x) = \begin{cases} 3x & if \quad x \leq -2 \\ x^2 - 1 & if \quad -2 < x < 2 \\ 3 & if \quad x \geq 2 \end{cases}$
Discuss continuity at $x = 2$ and $x = -2$

Q. 4 Solve the differential equation
$y - x\frac{dy}{dx} = \left(1 + x\frac{dy}{dx}\right)$

Q. 5 If $y = a\cos(\ln x) + b\sin\ln x$ Prove that
$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Q. 6 Find the area of the region bounded by the triangle whose sides are
$x - 2y - 6 = 0, \quad 3x - y + 3 = 0, \quad 2x + y - 4 = 0$

Q. 7 Maximize the function defined as; $f(x, y) = 2x + 3y$ subject to the constraints, $2x + y \leq 8$; $x + 2y \leq 14$; $x \geq 0$; $y \geq 0$.

Q. 8 Find the equation of the fangents to the ellipse $\frac{x^2}{128} + \frac{y^2}{8} = 1$ Which are parallel to the line $3x + 8y + 1 = 0$, also find the points of contact.

Q. 9 A particle is displaced from the point A (5, -5, -7) to the point B (6, 2, -2) under the action of constant force define by $10\underline{i} - \underline{j} + 11\underline{k}$. $4\underline{i} + 5\underline{j} + 9\underline{k}$ and $-2\underline{i} + \underline{j} - 9\underline{k}$ show that the total work done by the force is 102 units.