10th Class 2017

Math (Science)	Group-II	PAPER-II
Time: 20 Minutes	(Objective Type)	Max. Marks: 15

Note: Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1- **The solution set of equation $4x^2 - 16 = 0$ is:**
(a) $\{\pm 4\}$ (b) $\{4\}$
(c) $\{\pm 2\}$ ✓ (d) ± 2

2- **Cube roots of -1 are:**
(a) $-1, -\omega, -\omega^2$ ✓ (b) $-1, \omega, -\omega^2$
(c) $-1, -\omega, \omega^2$ (d) $1, -\omega, -\omega^2$

3- **If α, β are the roots of $x^2 - x - 1 = 0$, then product of the roots 2α and 2β is:**
(a) -2 (b) 2
(c) 4 (d) -4 ✓

4- **Find x in proportion $4 : x :: 5 : 15$:**
(a) $\frac{75}{4}$ (b) $\frac{4}{3}$
(c) $\frac{3}{4}$ (d) 12 ✓

5- **The third proportional of x^2 and y^2 is:**
(a) $\frac{y^2}{x^2}$ (b) x^2y^2
(c) $\frac{y^4}{x^2}$ ✓ (d) $\frac{y^2}{x^4}$

6- **The set having only one element is called:**
(a) Null set (b) Power set
(c) Singleton set ✓ (d) Subset

7- The different number of ways to describe a set is:
(a) 1 (b) 3 √
(c) 2 (d) 4

8- If $A \subseteq B$, then $A - B$ is equal to:
(a) A (b) ϕ √
(c) B (d) $B - A$

9- The extent of variation between two extreme observations of a data set is measured by:
(a) Range √ (b) Average
(c) Quartiles (d) Median

10- $\frac{3\pi}{4}$ radian = --- :
(a) 115° (b) 150°
(c) 30° (d) 135° √

11- The distance of any point of the circle to its centre is called:
(a) Diameter (b) A chord
(c) Radius √ (d) An arc

12- Tangents drawn at the ends of diameter of a circle are ---- to each other.
(a) Parallel √ (b) Non-parallel
(c) Collinear (d) Perpendicular

13- $\sec^2\theta$ = ---- :
(a) $1 - \sin^2\theta$ (b) $1 + \tan^2\theta$ √
(c) $1 + \cos^2\theta$ (d) $1 - \tan^2\theta$

14- The portion of a circle between two radii and an arc is called:
(a) Sector √ (b) Segment
(c) Chord (d) Diameter

15- How many common tangents can be drawn for two touching circles:
(a) 2 (b) 1
(c) 4 (d) 3 √

10th Class 2017

Math (Science)	Group-II	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: 12

(i) Define radical equation.

Ans An equation involving expression under the radical sign is called a radical equation.

(ii) Write the equation in standard form:

$$\frac{x}{x+1}+\frac{x+1}{x}=6$$

Ans Multiplying both sides by $x(x + 1)$

$$x(x+1)\frac{x}{x+1}+x(x+1)\frac{x+1}{x}=6x(x+1)$$

$$x(x)+(x+1)(x+1)=6x(x)+6x(1)$$

$$x^2+x^2+1+2x=6x^2+6x$$

$$2x^2+2x+1=6x^2+6x$$

$$0=6x^2+6x-2x^2-2x-1$$

$$0=4x^2+4x-1$$

$$\Rightarrow \quad 4x^2+4x-1=0$$

(iii) Define simultaneous equations.

Ans A system of equations having a common solution is called a system of simultaneous equations.

(iv) Evaluate: $(9+4\omega+4\omega^2)^3$

Ans Given, $(9+4\omega+4\omega^2)^3$

$$=(9+4(\omega+\omega^2))^3$$

$$=(9+4(-1))^3$$

$$=(9-4)^3$$

$$=5^3$$

$$=125$$

(v) Without solving, find the sum and the product of the roots of quadratic equation:

$$(l + m)x^2 + (m + n)x + n - l = 0$$

Ans Here: $a = l + m$, $b = m + n$, $c = n - l$

Let α, β be the roots of equation:

Sum of the roots:

$$S = \alpha + \beta = \frac{-b}{a} = -\frac{m + n}{l + m}$$

Product of the roots:

$$P = \alpha\beta = \frac{c}{a} = \frac{n - l}{l + m}$$

(vi) Use synthetic division to find the quotient and the remainder, when $(x^2 + 7x - 1) \div (x + 1)$.

Ans Let, $P(x) = x^2 + 7x - 1$

Here, $x - a = x + 1$

$$x - x - 1 = a$$

$$-1 = a$$

$$\Rightarrow \quad \boxed{a = -1}$$

By synthetic division

$$\begin{array}{r|rr|r} -1 & 1 & 7 & -1 \\ & & -1 & -6 \\ \hline & 1 & 6 & -7 \end{array}$$

The depressed equation is

$$x + 6 = 0$$

$\therefore$ Quotient $= Q(x) = x + 6$

and Remainder $= -7$

(vii) Define direct variation.

Ans If two quantities are related in such a way that increase (decrease) in one quantity causes increase (decrease) in the other quantity, then this variation is called direct variation.

(viii) Find fourth proportional: $4x^4, 2x^3, 18x^5$

Ans Let y be the fourth proportional, then

$$4x^4 : 2x^3 :: 18x^5 : y$$

Product of extremes = Product of means

$$(4x^4)\,y = (2x^3)(18x^5)$$

$$y = \frac{(2x^3)(18x^5)}{4x^4}$$

$$\boxed{y = 9x^4}$$

(ix) **If $3(4x - 5y) = 2x - 7y$, find the ratio $x : y$.**

Ans

$$3(4x - 5y) = 2x - 7y$$
$$12x - 15y = 2x - 7y$$
$$12x - 2x = -7y + 15y$$
$$10x = 8y$$
$$\frac{x}{y} = \frac{8}{10}$$
$$\frac{x}{y} = \frac{4}{5}$$
$$x : y = 4 : 5$$

3. Write short answers to any SIX (6) questions: 12

(i) **Define a rational fraction.**

Ans An expression of the form $\frac{N(x)}{D(x)}$, where $N(x)$ and $D(x)$ are polynomial in x with real coefficients and $D(x) \neq 0$, is called a rational fraction.

(ii) **How can we make partial fractions of $\frac{7x-9}{(x+1)(x-3)}$.**

Ans Let

$$\frac{7x-9}{(x+1)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(x-3)} \qquad \text{(i)}$$

Multiplying by $(x + 1)(x - 3)$, we get

$$7x - 9 = A(x - 3) + B(x + 1)$$
$$7x - 9 = Ax - 3A + Bx + B$$
$$7x - 9 = Ax + Bx - 3A + B$$
$$7x - 9 = (A + B)\,x - 3A + B$$

By comparing coefficients of x and constant terms

$$A + B = 7 \quad \text{(ii)}$$
$$-3A + B = -9 \quad \text{(iii)}$$

By subtracting (iii) from (ii), gives

$$\begin{array}{r} A + B = 7 \\ \mp 3A \pm B = \mp 9 \\ \hline 4A \quad = 16 \end{array}$$

$$A = \frac{16}{4}$$
$$A = 4$$

Put $A = 4$ in (ii)

$$4 + B = 7$$
$$B = 7 - 4$$
$$B = 3$$

By putting values of A, B in (i), we get

$$\frac{7x - 9}{(x + 1)(x - 3)} = \frac{4}{(x + 1)} + \frac{3}{(x + 3)}$$

(iii) Define complement of a set.

Ans If U is a universal set and A is subset of U, then the complement of A is the set of those elements of U which are not contained in A and is denoted by A^c or A'.

(iv) Find a and b if $(a - 4, b - 2) = (2, 1)$.

Ans

$$a - 4 = 2 \quad ; \quad b - 2 = 1$$
$$a = 2 + 4 \quad ; \quad b = 1 + 2$$
$$\boxed{a = 6} \quad ; \quad \boxed{b = 3}$$

(v) Define domain and range of a relation.

Ans **Domain:**

Domain of relation denoted by DomR is the set consisting of all the first elements of each ordered pair in the relation.

Range:

Range of relation denoted by range R is the set consisting of all the second elements of each ordered pair in the relation.

(vi) **Find $A \cap B$ if $A = \{2, 3, 5, 7\}$ and $B = \{3, 5, 8\}$.**

Ans $A \cap B = \{2, 3, 5, 7\} \cap \{3, 5, 8\}$
$= \{3, 5\}$

(vii) **The marks of seven students in Mathematics are as follows. Find Arithmetic Mean: 45, 60, 74, 58, 65, 63, 49.**

Ans Let, X = Marks of students
$X = 45, 60, 74, 58, 65, 63, 49$

The Arithmetic Mean: $(\bar{X})$

$$\bar{X} = \frac{\Sigma X}{n}$$

$$= \frac{45 + 60 + 74 + 58 + 65 + 63 + 49}{7}$$

$$= \frac{414}{7}$$

$$\boxed{\bar{X} = 59.14}$$

(viii) **Find geometric mean of 2, 4 and 8.**

Ans
$$G.M = (2 \times 4 \times 8)^{1/3}$$
$$= (64)^{1/3}$$
$$= (4^3)^{1/3}$$
$$= 4$$

(ix) **Define mode.**

Ans Mode is defined as the most frequent occurring observation in the data.

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4. Write short answers to any SIX (6) questions: 12

(i) **Define radian.**

Ans The angle subtended at centre of the circle by an arc, whose length is equal to the radius of the circle is called one radian.

(ii) **Express 225° into radian.**

Ans
$$225^\circ = 225 \times \frac{\pi}{180}$$

$$= \frac{5\pi}{4} \text{ radians}$$

(iii) **In a circle of radius 12 m, find the length of an arc which subtends a central angle $\theta = 1.5$ radian.**

Ans

$$r = 12\text{ m}$$
$$\theta = 1.5\text{ radian}$$
$$l = ?$$
$$l = r\theta$$
$$= (12)(1.5)$$
$$l = 18\text{ m}$$

(iv) **Define projection of a point.**

Ans The projection of a given point on a line segment is the foot of perpendicular drawn from the point on that line segment.

(v) **Define radial segment.**

Ans Radical segment of a circle is a line segment, determined by the centre and a point on a circle.

(vi) **Define the tangent to a circle.**

Ans A tangent to a circle is the straight line which touches the circumference at a single point only.

(vii) **Define sector of a circle.**

Ans The circular region bounded by an arc of a circle and its two corresponding radical segments is called a sector of a circle.

(viii) **Define central angle.**

Ans The angle subtended by an arc at the centre of a circle is called its central angle.

(ix) **Define geometry.**

Ans Geometry is an important branch of mathematics, which deals with the shape, size and position of geometric figures.

(Part-II)

NOTE: Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve the equation: $2x + 5 = \sqrt{7x + 16}$ (4)

Ans Given equation:

$$2x + 5 = \sqrt{7x + 16}$$

Taking square on both sides,

$$(2x + 5)^2 = (\sqrt{7x + 16})^2$$
$$(2x)^2 + (5)^2 + 2(2x)(5) = 7x + 16$$
$$4x^2 + 25 + 20x = 7x + 16$$
$$4x^2 + 20x + 25 - 7x - 16 = 0$$
$$4x^2 + 13x + 9 = 0$$
$$4x^2 + 4x + 9x + 9 = 0$$
$$4x(x + 1) + 9(x + 1) = 0$$
$$(x + 1)(4x + 9) = 0$$
$$x + 1 = 0 \quad ; \quad 4x + 9 = 0$$
$$\boxed{x = -1} \quad ; \quad 4x = -9$$
$$\boxed{x = \frac{-9}{4}}$$

(b) Use synthetic division to find the values of l and m, if $(x + 3)$ and $(x - 2)$ are the factors of the polynomial $x^3 + 4x^2 + 2lx + m$. **(4)**

Ans Here $P(x) = x^3 + 4x^2 + 2lx + m$

and $x - a = x + 3$

$\Rightarrow$ $\boxed{a = -3}$

$$\begin{array}{r|cccc}
 & 1 & 4 & 2l & m \\
-3 & & -3 & -3 & -6l + 9 \\
\hline
 & 1 & 1 & 2l - 3 & -6l + m + 9
\end{array}$$

Since -3 is zero of polynomial, so remainder equal to zero.

$-6l + m + 9 = 0$ (1)

Again,

$$x - a = x - 2$$
$$-a = -2$$
$$a = 2$$

Again using synthetic division,

$$\begin{array}{r|cccc}
 & 1 & 4 & 2l & m \\
2 & & 2 & 12 & 4l + 24 \\
\hline
 & 1 & 6 & 2l + 12 & 4l + m + 24
\end{array}$$

Since 2 is zero of the polynomial, so remainder equal to zero.

$$4l + m + 24 = 0 \quad (2)$$

By solving equation (1) and equation (2),

$$\begin{array}{r} -6l + m + 9 = 0 \\ \underline{\underset{-}{+}4l \underset{-}{+} m \underset{-}{+} 24 = 0} \\ -10l - 15 = 0 \end{array}$$

$$-10l = 15$$

$$l = \frac{15}{-10}$$

$$l = \frac{-3}{2}$$

Put $l = \frac{-3}{2}$ in equation (1), we get

$$-6\left(\frac{-3}{2}\right) + m + 9 = 0$$

$$9 + m + 9 = 0$$

$$m + 18 = 0$$

$$\boxed{m = -18}$$

Thus $\boxed{l = \frac{-3}{2}}$, $\boxed{m = -18}$

Q.6.(a) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ (a, b, c, d, e, f, $\neq 0$), then show that $\frac{a}{b} = \sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}}$. (4)

Ans Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = K$

$$\frac{a}{b} = K, \quad \frac{c}{d} = K, \quad \frac{e}{f} = K$$

$$a = bK, \quad c = dK, \quad e = fK$$

$$\text{L.H.S} = \frac{a}{b}$$

$$= \frac{bK}{b}$$

$$= K \quad (1)$$

$$\text{R.H.S} = \sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}}$$

$$= \sqrt{\frac{(bK)^2 + (dK)^2 + (fK)^2}{b^2 + d^2 + f^2}}$$

$$= \sqrt{\frac{b^2K^2 + d^2K^2 + f^2K^2}{b^2 + d^2 + f^2}}$$

$$= \sqrt{\frac{K^2(b^2 + d^2 + f^2)}{b^2 + d^2 + f^2}}$$

$$= \sqrt{K^2} = K \quad (2)$$

From (1) and (2),

$$\text{L.H.S} = \text{R.H.S}$$

i.e., $$\frac{a}{b} = \sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}}$$ Proved

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(b) Resolve into partial fractions: $\dfrac{x - 11}{(x - 4)(x + 3)}$ **(4)**

Ans $$\frac{x - 11}{(x - 4)(x + 3)} = \frac{A}{x - 4} + \frac{B}{x + 3}$$

$$x - 11 = A(x + 3) + B(x - 4) \quad (i)$$

Put $x = 4$, $x = -3$ in (i)

Firstly,

$$4 - 11 = A(4 + 3) + B(4 - 4)$$

$$-7 = A(7) + 0$$

$$\Rightarrow \quad 7A = -7$$

$$\boxed{A = -1}$$

And

$$-3 - 11 = A(-3 + 3) + B(-3 - 4)$$

$$-14 = 0 + B(-7)$$

$$\Rightarrow \quad -7B = -14$$

$$\boxed{B = 2}$$

So,

$$\frac{x - 11}{(x - 4)(x + 3)} = \frac{-1}{x - 4} + \frac{2}{x + 3}$$

Q.7.(a) If $U = \{1, 2, 3, 4, ---, 10\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{1, 4, 7, 10\}$ then prove that $B - A = B \cap A'$. (4)

Ans To show $B - A = B \cap A'$

$$\text{L.H.S} = B - A$$
$$= \{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\}$$
$$= \{4, 10\} \quad (1)$$

Now,

$$A' = U - A$$
$$= \{1, 2, 3, 4, ----, 10\} - \{1, 3, 5, 7, 9\}$$
$$= \{2, 4, 6, 8, 10\}$$

and

$$\text{R.H.S} = B \cap A'$$
$$= \{1, 4, 7, 10\} \cap \{2, 4, 6, 8, 10\}$$
$$= \{4, 10\} \quad (2)$$

From equation (1) and equation (2),

$$\text{L.H.S} = \text{R.H.S}$$
$$B - A = B \cap A'$$

(b) The marks of six students in mathematics are as follows. Determine variance: (4)

Students	1	2	3	4	5	6
Marks	60	70	30	90	80	42

Ans

X	X^2
60	3600
70	4900
30	900
90	8100
80	6400
42	1764
372	25664

$$\text{Variance} = S^2 = \frac{\Sigma X^2}{n} - \left(\frac{\Sigma X}{n}\right)^2$$

$$= \frac{25664}{6} - \left(\frac{372}{6}\right)^2$$
$$= 4277.33 - (62)^2$$
$$= 4277.33 - 3844$$
$$\boxed{S^2 = 433.33}$$

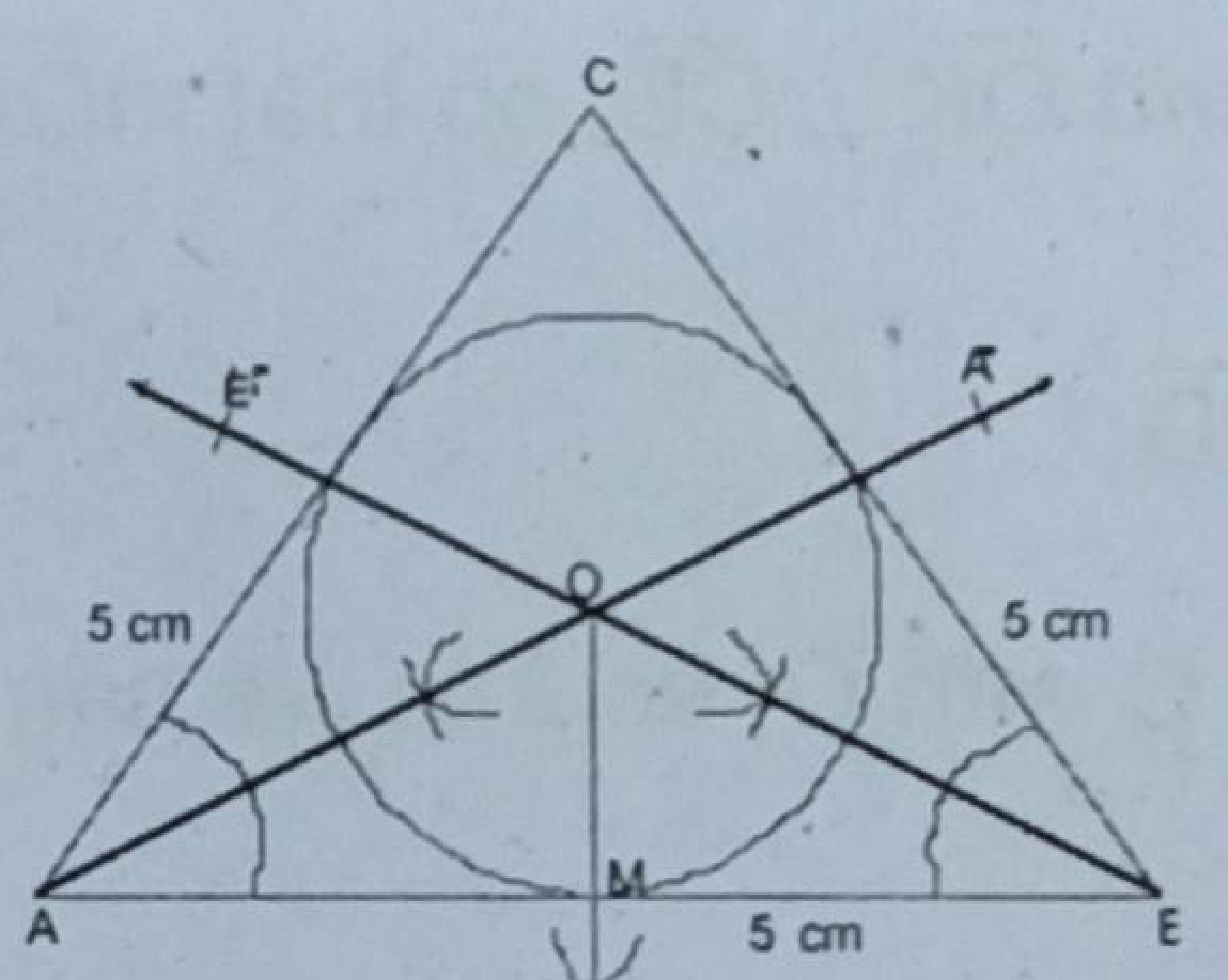
Q.8.(a) Prove that: $\frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta} = 4\tan\theta\sec\theta.$ **(4)**

Ans L.H.S $= \frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta}$

$$= \frac{(1+\sin\theta)^2 - (1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$$
$$= \frac{(1+\sin^2\theta + 2\sin\theta) - (1+\sin^2\theta - 2\sin\theta)}{(1)^2 - (\sin\theta)^2}$$
$$= \frac{1+\sin^2\theta + 2\sin\theta - 1 - \sin^2\theta + 2\sin\theta}{1-\sin^2\theta}$$
$$= \frac{4\sin\theta}{\cos^2\theta}$$
$$= 4\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}$$
$$= 4\tan\theta\sec\theta$$
$$= \text{R.H.S}$$

(b) Inscribe a circle in an equilateral triangle ABC with each side of length 5 cm. **(4)**

Ans



Steps of Construction:

(i) Draw a ΔABC with each side = 5 cm.

(ii) Draw $\overrightarrow{AA'}$ bisector of $\angle A$.

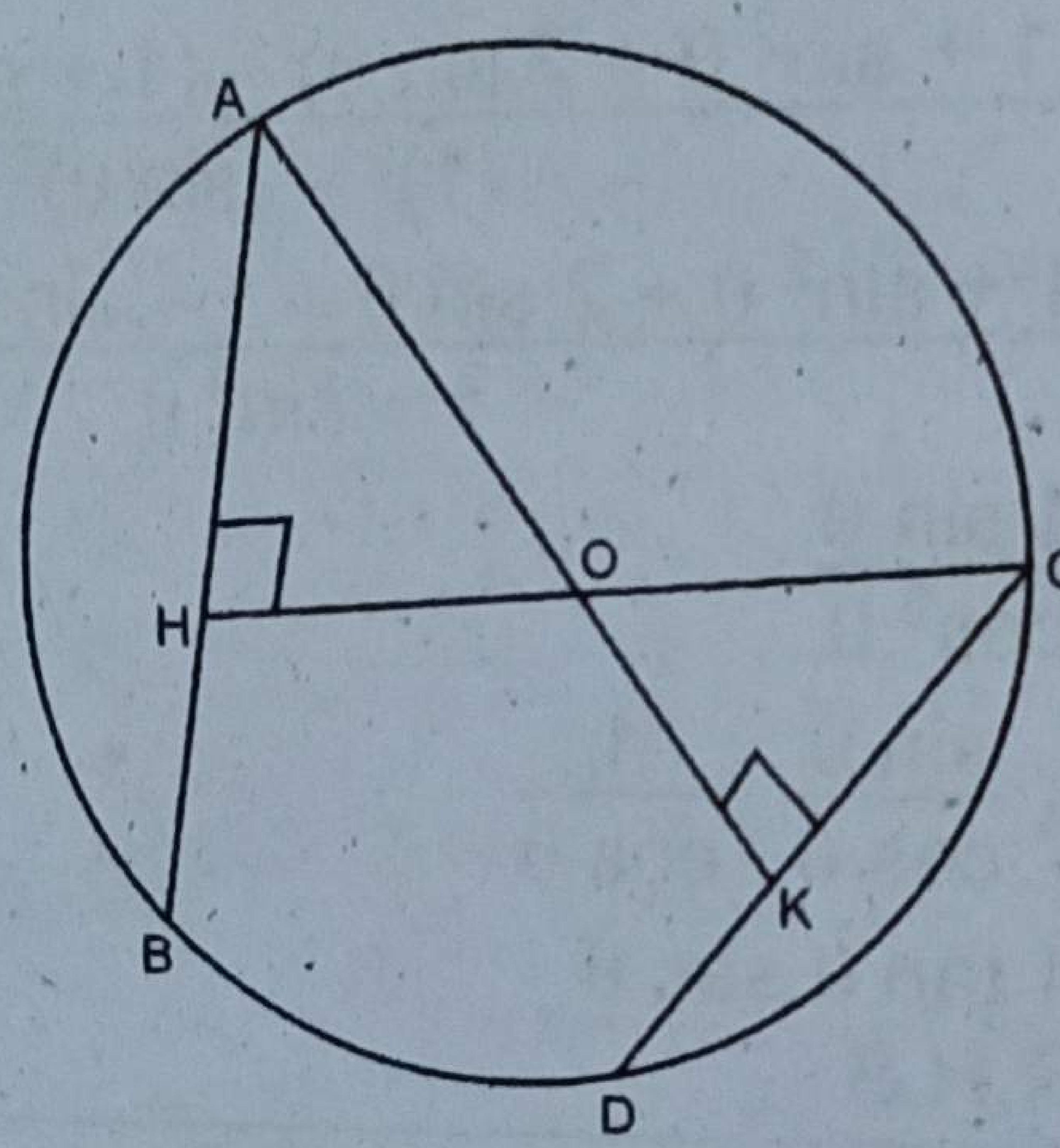
(iii) Draw BE bisecter of $\angle B$.
$\overrightarrow{AA'}$ and $\overrightarrow{BE}$ intersect at point O.

(iv) Drop $\overline{OM} \perp \overline{AB}$.

(v) Take O as centre and draw a circle with $m\overline{OM}$ as radius.
This is inscribed circle to triangle ABC.

Q.9. Prove that two chords of a circle which are equidistant from the centre, are congruent. (4)

Ans



Given:

$\overline{AB}$ and $\overline{CD}$ are two chords of a circle with center O.

$\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$, so that $m\overline{OH} = m\overline{OK}$.

To prove:

$m\overline{AB} = m\overline{CD}$

Construction:

Join A and C with O, so that we can form $\angle$rt Δ^s OAH and OCK.

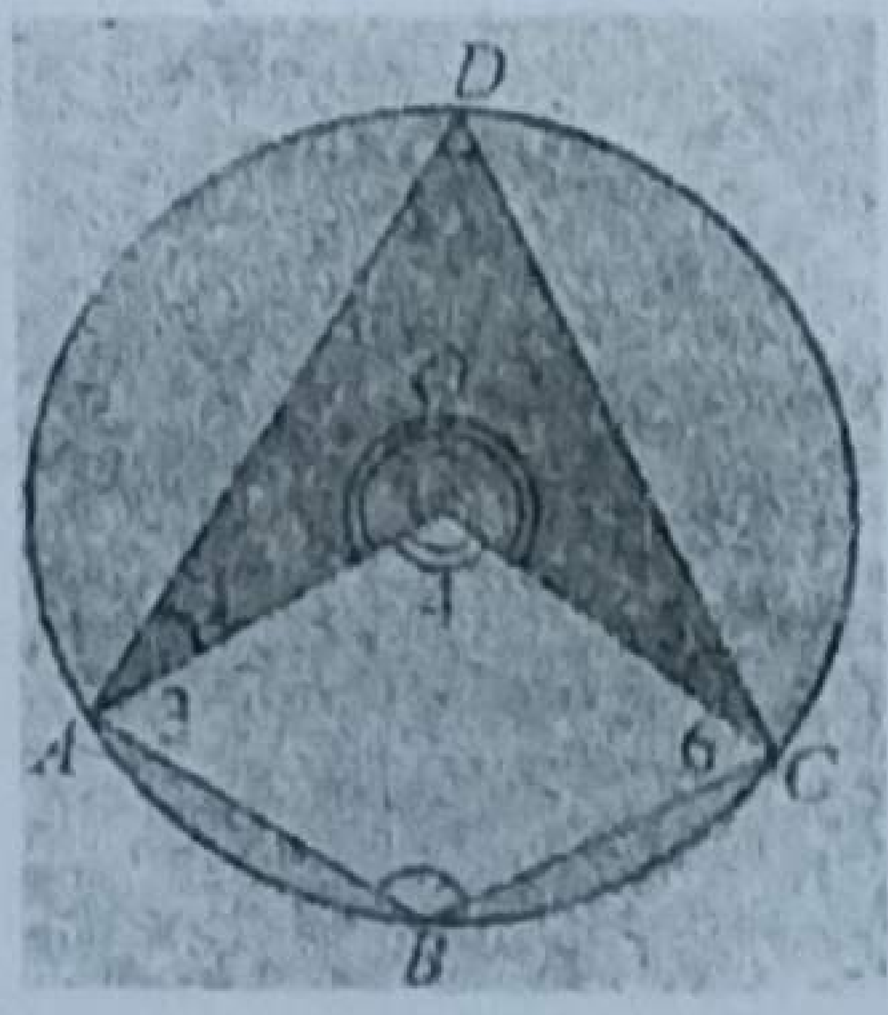
Proof:

Statements		Reasons
In $\angle$rt Δ^s OAH $\leftrightarrow$ OCK		
$\because$ hyp. $\overline{OA}$ = hyp. $\overline{OC}$		Radii of the same circle
$m\overline{OH} = m\overline{OK}$		Given
$\therefore$ $\Delta OAH \cong \Delta OCK$		H.S postulate
So		
$m\overline{AH} = m\overline{CK}$	(i)	Corresponding sides of congruent triangles
But		
$m\overline{AH} = \frac{1}{2} m\overline{AB}$	(ii)	$\overline{OH} \perp$ chord $\overline{AB}$ (Given)
Similarly,		
$m\overline{CK} = \frac{1}{2} m\overline{CD}$	(iii)	$\overline{OK} \perp$ chord $\overline{CD}$ Given
Since $m\overline{AH} = m\overline{CK}$		Already proved in (i)
$\therefore$ $\frac{1}{2} m\overline{AB} = \frac{1}{2} m\overline{CD}$		Using (ii) and (iii)
or		
$m\overline{AB} = m\overline{CD}$		

OR

Prove that the opposite angles of any quadrilateral inscribed in a circle are supplementary.

Ans



Given:

ABCD is a quadrilateral inscribed in a circle with centre O.

To prove:

$$\begin{cases} m\angle A + m\angle C = 2\angle rts \\ m\angle B + m\angle D = 2\angle rts \end{cases}$$

Construction:

Draw $\overline{OA}$ and $\overline{OC}$.
Write $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$ as shown in the figure.

Statements	Reasons
Standing on the same arc ADC, $\angle 2$ is a central angle. Whereas $\angle B$ is the circumangle	Arc ADC of the circle with centre O.
$\therefore \quad m\angle B = \frac{1}{2}(m\angle 2)$ (i)	By theorem 1
Standing on the same arc ABC, $\angle 4$ is a central angle whereas $\angle D$ is the circumangle	Arc ABC of the circle with centre O.
$\therefore \quad m\angle D = \frac{1}{2}(m\angle 4)$ (ii)	By theorem 1
$\Rightarrow \quad m\angle B + m\angle D = \frac{1}{2} m\angle 2 + \frac{1}{2} m\angle 4$ $= \frac{1}{2}(m\angle 2 + m\angle 4) = \frac{1}{2}$ (Total central angle)	Adding (i) and (ii)
i.e., $m\angle B + m\angle D = \frac{1}{2}(4\angle rt)$ $= 2\angle rt$	
Similarly, $m\angle A + m\angle C = 2\angle rt$	