10th Class 2015

Math (Science)	Group-I	
Time: 20 Minutes	(Objective Type)	Max. Marks: 15

Note: Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1- A set with no element is called:

(a) Subset (b) Empty set √
(c) Singleton set (d) Super set

2- A histogram is a set of adjacent -----:

(a) Squares (b) Rectangles √
(c) Circles (d) None of these

3- $\sec^2\theta$ = -----:

(a) $1 - \sin^2\theta$ (b) $1 + \tan^2\theta$ √
(c) $1 + \cos^2\theta$ (d) $1 - \tan^2\theta$

4- Two square roots of unity are:

(a) ω, ω^2 (b) $1, -\omega$
(c) $1, \omega$ (d) $1, -1$ √

5- Product of cube roots of unity is:

(a) 0 (b) 1 √
(c) –1 (d) 3

6- In a proportional a : b : : c : d , a and d are called:

(a) Means (b) Extremes √
(c) Fourth proportional (d) None of these

7- A circle has only one -----:

(a) Secant (b) Chord
(c) Diameter (d) Centre √

8- Mean is affected by change in -----:

(a) Place (b) Scale
(c) Rate (d) Origin √

9- Number of terms in a quadratic equation is:

(a) 1 (b) 2
(c) 3 ✓ (d) 4

10- A set having only one element is called:

(a) Empty set (b) Power set
(c) Singleton set ✓ (d) Subset

11- Out of two congruent arcs of a circle if one arc makes central angle of 30°, then the other arc will subtend the central angle of -----:

(a) 15° (b) 30° ✓
(c) 45° (d) 60°

12- The chord passing through the centre of a circle is called:

(a) Radius (b) Diameter ✓
(c) Circumference (d) Tangent

13- The tangent and radius of a circle at the point of contact are -----:

(a) Parallel (b) Not perpendicular
(c) Perpendicular ✓ (d) None of these

14- In a proportional a : b : : c : d, b and c are called:

(a) Means ✓ (b) Extremes
(c) Fourth proportional (d) None of these

15- Roots of the equation $4x^2 - 5x + 2 = 0$ are:

(a) Irrational (b) Imaginary ✓
(c) Rational (d) None of these

10th Class 2015

Math (Science)	Group-I	
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: 12

(i) Define exponential equation.

Ans In exponential equations, variable occurs in exponent. For example, $5^{1+x} + 5^{1-x} = 26$

(ii) Solve by factorization: $5x^2 = 15$

Ans $5x^2 - 15x = 0$

$5x(x - 3) = 0$

From the above equation:

$5x = 0$ $\qquad$ $x - 3 = 0$

$\boxed{x = 0}$ $\qquad$ $\boxed{x = 3}$

Thus, solution set: $\{0, 3\}$.

(iii) Find the value of: $\omega^{37} + \omega^{38} + 1$

Ans Given $\omega^{37} + \omega^{38} + 1$

$= \omega^{36+1} + \omega^{36+2} + 1$

$= \omega^{36}.\omega + \omega^{36}.\omega^2 + 1$

$= (\omega^3)^{12}\omega + (\omega^3)^{12}.\omega^2 + 1$

$= (1)^{12}\omega + (1)^{12}\omega^2 + 1$

$= (1)\omega + (1)\omega^2 + 1$

$= \omega + \omega^2 + 1$

As we know that $\omega + \omega^2 + 1 = 0$

Thus, the answer is zero.

(iv) If α, β are the roots of the equation $2x^2 + 3x + 4 = 0$, then find the value $\frac{1}{\alpha} + \frac{1}{\beta}$.

Ans $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$$

$$\frac{1}{\alpha}+\frac{1}{\beta}=(\alpha+\beta)\frac{1}{\alpha\beta} \qquad (A)$$

From equation $2x^2 + 3x + 4 = 0$, we have

$$\alpha+\beta=\frac{-b}{a}=\frac{-3}{2}$$

$$\alpha\beta=\frac{c}{a}=\frac{4}{2}=2$$

By putting these values in equation 'A',

$$\frac{1}{\alpha}+\frac{1}{\beta}=\left(\frac{-3}{2}\right)\left(\frac{1}{2}\right)$$

$$=\frac{-3}{4}$$

(v) Find the discriminant of: $x^2 - 5x + 5 = 0$

Ans Compare the above equation with general quadratic equation $ax^2 + bx + c = 0$, we get

$a = 1$

$b = -5$

$c = 5$

Discriminant $= b^2 - 4ac$

$= (-5)^2 - 4(1)(5)$

$= 25 - 20$

$= 5 > 0$

(vi) Write the quadratic equation having the roots: –1, –7.

Ans As we have, –1 and –7, the roots of required quadratic equation, so

Addition of roots = S = (–1) + (–7) = –1 – 7 = –8

Multiplication of roots = P = (–1)(–7) = 7

Thus, quadratic equation is

$x^2 - Sx + P = 0$

$x^2 - (-8)x + 7 = 0$

$x^2 + 8x + 7 = 0$

(vii) Define proportion and give example.

Ans A proportion is a statement, which is expressed as an equivalence of two ratios. If two ratios a : b and c : d are equal, then we can write a : b = c : d. Where quantities a, d are called extremes, while b, c are called means.

Symbolically, the proportion of a, b, c and d is written as:

$$a : b :: c : d$$

or $$a : b = c : d$$

or $$\frac{a}{b} = \frac{c}{d}, \quad \textit{i.e.,} \quad ad = bc$$

Example:

Find x, if 60 m : 90 m : : 20 Kg : x Kg

$$60 : 90 = 20 : x$$

$$\frac{60}{90} = \frac{20}{x}$$

$$60x = 20 \times 90$$

$$x = \frac{20 \times 90}{60}$$

$$x = 30 \text{ Kg}$$

(viii) If $3(4x - 5y) = 2x - 7y$, then find x : y.

Ans Given:

$$3(4x - 5y) = 2x - 7y$$

$$12x - 15y = 2x - 7y$$

$$12x - 2x = 15y - 7y$$

$$10x = 8y$$

Dividing both sides by 2,

$$\frac{10x}{2} = \frac{8y}{2}$$

$$5x = 4y$$

$$\frac{x}{y} = \frac{4}{5}$$

Converting the above equation in ratios

$$x : y = 4 : 5$$

(ix) Find the third proportional to: $a^2 - b^2$, $a - b$

Ans Let c is the third proportional

$$a^2 - b^2 : a - b :: a - b : c$$

Product of extremes = Product of means

$$(a^2 - b^2)(c) = (a - b)(a - b)$$

$$c = \frac{(a - b)(a - b)}{a^2 - b^2}$$

$$c = \frac{(a - b)(a - b)}{(a + b)(a - b)}$$

$$\boxed{c = \frac{a - b}{a + b}}$$

3. Write short answers to any SIX (6) questions: 12

(i) What is proper fraction?

Ans A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called a proper fraction if the degree of the polynomial N(x) in the numerator is less than the degree of the polynomial D(x) in the denominator.

For example,

$$\frac{2}{x + 1}, \frac{2x - 3}{x^2 + 4} \text{ and } \frac{3x^2}{x^3 + 1} \text{ are proper fractions.}$$

(ii) Convert into proper fraction: $\frac{x^2 + x + 1}{x^2 + 2}$

Ans

$$\begin{array}{r|l} & 1 \\ \hline x^2 + 2 & x^2 + x + 1 \\ & \pm x^2 \quad \pm 2 \\ \hline & x - 1 \end{array}$$

So,

$$\frac{x^2 + x + 1}{x^2 + 2} = 1 + \frac{x - 1}{x^2 + 2}$$

(iii) Define intersection of two sets.

Ans The intersection of two sets, let A and B written as $A \cap B$ (read as 'A intersection B') is the set consisting of all the common elements of A and B. Thus,

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

Clearly $x \in A \cap B \Rightarrow x \in A$ and $x \in B$.

(iv) If $x = \{1, 4, 7, 9\}$ and $y = \{2, 4, 5, 9\}$, then find $x - y$ and $y - x$.

Ans Given $x = \{1, 4, 7, 9\}$

$y = \{2, 4, 5, 9\}$

So,

$$x - y = \{1, 4, 7, 9\} - \{2, 4, 5, 9\}$$
$$x - y = \{1, 7\}$$

And

$$y - x = \{2, 4, 5, 9\} - \{1, 4, 7, 9\}$$
$$y - x = \{2, 5\}$$

(v) If $A = \{0, 2, 4\}$, $B = \{-1, 3\}$, then find $A \times A$ and $B \times B$.

Ans Given $A = \{0, 2, 4\}$

$B = \{-1, 3\}$

$$A \times A = \{0, 2, 4\} \times \{0, 2, 4\}$$
$$= \{(0, 0), (0, 2), (0, 4), (2, 0), (2, 2), (2, 4), (4, 0), (4, 2), (4, 4)\}$$

$$B \times B = \{-1, 3\} \times \{-1, 3\}$$
$$B \times B = \{(-1, -1\}, (-1, 3), (3, -1), (3, 3)\}$$

(vi) If set M has 5 numbers, then find the number of binary relations in M?

Ans Number of elements in $M = 5$

Number of binary relations in $M = 2^{5 \times 5}$

$$= 2^{25}$$

(vii) Define Geometric Mean.

Ans Geometric mean of a variable X is the n^{th} positive root of the product of the $x_1, x_2, x_3, \ldots\ldots, x_n$ observations. In symbols, we write

$$G.M = (x_1, x_2, x_3, \ldots\ldots x_n)^{1/n}$$

The above formula can also be written by using logarithm.

$$G.M = \text{Antilog}\left(\frac{\Sigma \log x}{n}\right) \qquad \text{[For ungrouped data]}$$

$$G.M = \text{Antilog}\left(\frac{\Sigma f \log x}{\Sigma f}\right)$$ [For grouped data]

(viii) Find the Standard Deviation for the data 12, 6, 7, 3, 2.

Ans Firstly arrange the values and make table for calculate standard deviation

X	X^2
2	4
3	9
6	36
7	49
12	144
30	242

Formula for Standard Deviation

$$S.D(x) = S = \sqrt{\left[\left(\frac{\Sigma x^2}{n}\right) - \left(\frac{\Sigma x}{n}\right)^2\right]}$$

$$S = \sqrt{\left(\frac{242}{5}\right) - \left(\frac{30}{5}\right)^2}$$

$$S = \sqrt{48.4 - \frac{900}{25}}$$

$$S = \sqrt{48.4 - 36}$$

$$S = \sqrt{12.4}$$

$$\boxed{S = 3.52}$$

(ix) Find the Harmonic Mean for data 10, 5, 9, 6.

Ans By arranging the data and making table below:

X	$\frac{1}{X}$
5	0.2
6	0.1667
9	0.1111
10	0.1
	0.5778

Formula for Harmonic Mean:

$$H.M = \frac{n}{\Sigma(\frac{1}{x})}$$

Here n = 4, (number of observations)

$$H.M = \frac{4}{0.5778}$$

$$\boxed{H.M = 6.923}$$

4. Write short answers to any SIX (6) questions: 12

(i) Define an angle.

Ans An angle is defined as the union of two non-collinear rays with same common end point. The rays are called arms of the angle and the common end point is known as vertex of the angle.

(ii) Find r when l = 56 cm and $\theta = 45^\circ$.

Ans As $\theta = 45^\circ = 45 \times 1^\circ$

$$= 45 \times \frac{\pi}{180^\circ}$$

$$= \frac{\pi}{4}$$

Radian : $\theta = 0.7854$

As we known that

$$l = r\theta$$

$$r = \frac{l}{\theta}$$

$$r = \frac{56}{0.7854}$$

$$\boxed{r = 71.30 \text{ cm}}$$

(iii) Convert the following into degree: $\frac{3\pi}{4}$

Ans $\frac{3\pi}{4} = \frac{3\pi}{4}$ Radian

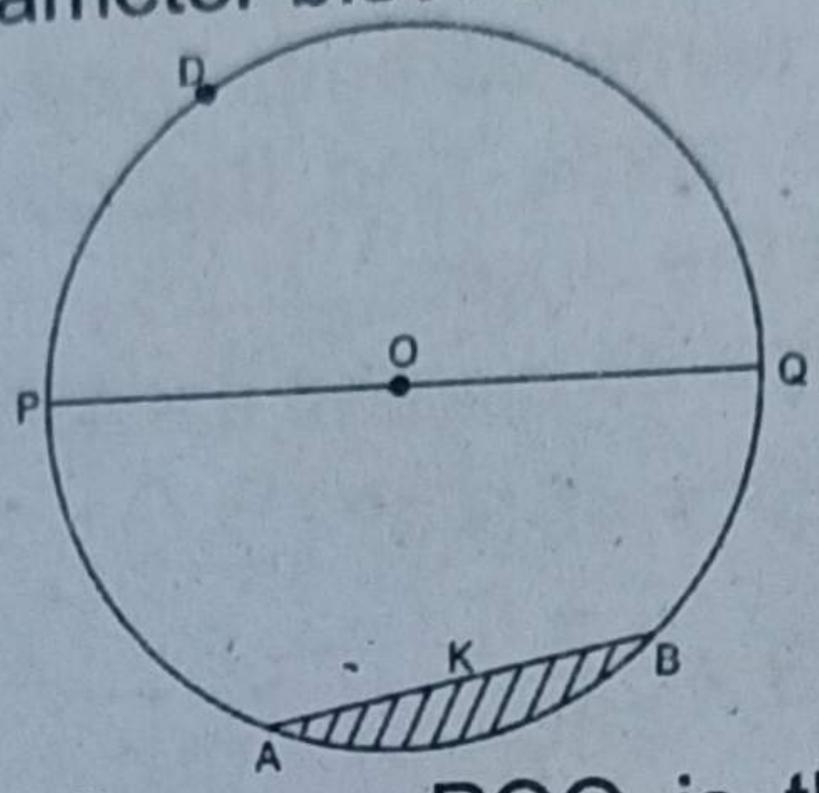
$$= \frac{3\pi}{4} \times 1 \text{ Radian}$$

$$= \frac{3\pi}{4} \times \frac{180^\circ}{\pi}$$

$$= 135^\circ$$

(iv) Differentiate between chord and the diameter of a circle.

Ans A chord of a circle is a line segment joining any two points A and B on the circumference of a circle, whereas diameter is the chord passing through the center of a circle. Evidently, diameter bisects a circle.



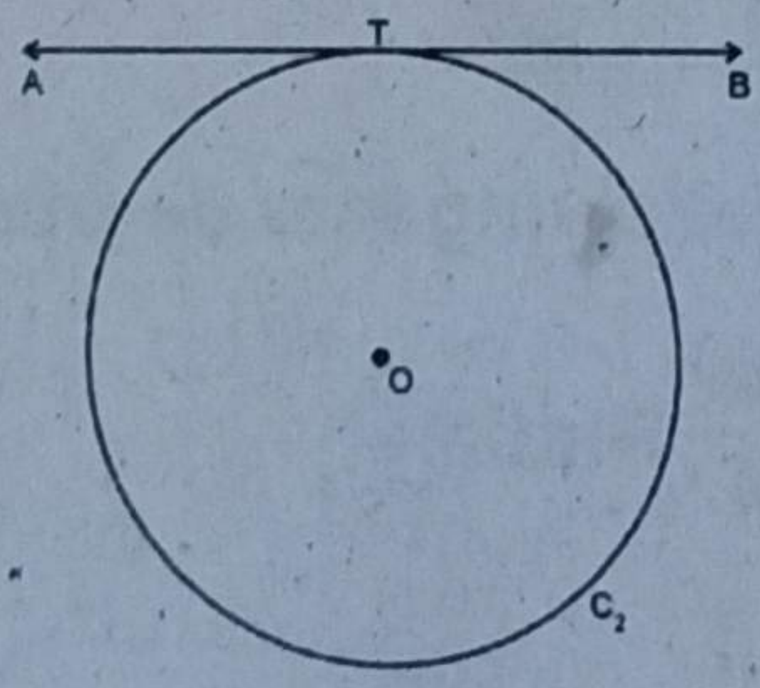
In the above diagram, POQ is the diameter, while AKB is only the chord of the circle.

(v) Write the formula to find the area of a circle.

Ans Area of a circle = πr^2

(vi) Define the tangent of a circle.

Ans A tangent to a circle is the straight line which touches the circumference at a single point only. The point of tangency is also known as the point of contact. In the figure $\overleftrightarrow{AB}$ indicates the tangent line to the circle C_2.

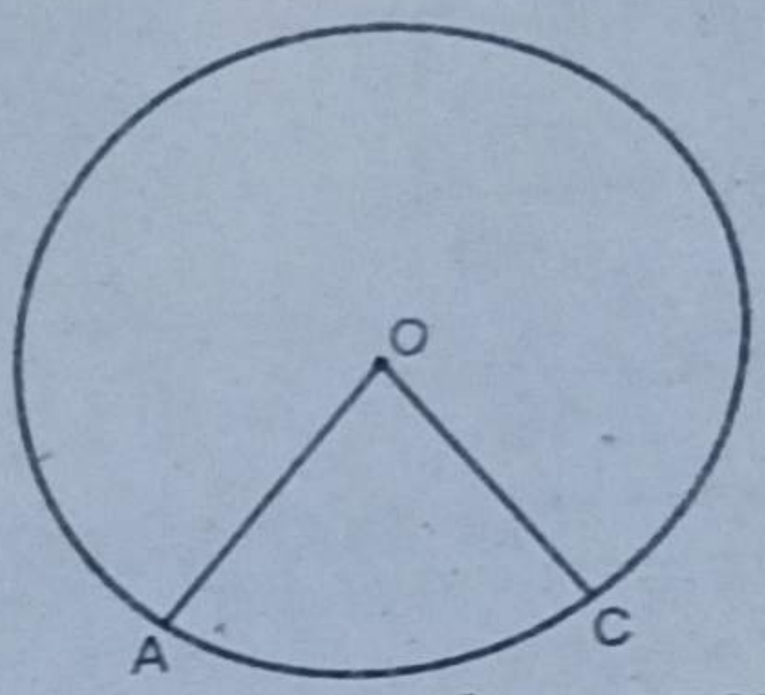


(vii) Define the circumference of a circle.

Ans The length of the boundary of the circle is called the circumference. It is calculated by $2\pi r$.

(viii) Define the central angle and form the figure.

Ans $\angle AOC$ is the central angle of the circle whose vertex is at the centre O and its arms meet at the end points of arc AC.



(ix) Differentiate between the escribed circle and circumscribed circle.

Ans The circle passing through the vertices of the triangle is called circumscribed circle. While a circle touches one side of a triangle externally and the other two produced sides internally, is called escribed circle.

(Part-II)

NOTE: Attempt any Three (3) questions.

Q.5.(a) Solve the equation by completing square: (4)

$$7x^2 + 2x - 1 = 0$$

Ans As given

$$7x^2 + 2x - 1 = 0 \qquad \text{(i)}$$

$$7x^2 + 2x = 1$$

Dividing both sides by '7'

$$\frac{7x^2}{7} + \frac{2x}{7} = \frac{1}{7}$$

$$x^2 + \frac{2x}{7} = \frac{1}{7} \qquad \text{(ii)}$$

Adding both sides with $\left(\frac{1}{7}\right)^2$

$$x^2 + \frac{2x}{7} + \left(\frac{1}{7}\right)^2 = \frac{1}{7} + \left(\frac{1}{7}\right)^2$$

$$(x)^2 + 2(x)\left(\frac{1}{7}\right) + \left(\frac{1}{7}\right)^2 = \left(\frac{1}{7}\right) + \left(\frac{1}{7}\right)^2$$

$$(x + \frac{1}{7})^2 = \frac{1}{7} + \frac{1}{49}$$

$$(x + \frac{1}{7})^2 = \frac{7 + 1}{49}$$

$$(x + \frac{1}{7})^2 = \frac{8}{49}$$

By taking under root, both sides

$$\sqrt{(x + \frac{1}{7})^2} = \pm\sqrt{\frac{8}{49}}$$

$$x + \frac{1}{7} = \pm\frac{2\sqrt{2}}{7}$$

$$x = \frac{-1}{7} \pm \frac{2\sqrt{2}}{7}$$

$$x = \frac{-1 \pm 2\sqrt{2}}{7}$$

Thus, solution set is $\left\{\frac{-1 \pm 2\sqrt{2}}{7}\right\}$.

(b) Find p, if the sum of the squares of the roots of the equation $4x^2 + 3px + p^2 = 0$ is unity. (4)

Ans If α, β are the roots of the equation $4x^2 + 3px + p^2 = 0$, then

$$\alpha + \beta = \frac{-b}{a} = \frac{-3p}{4}$$

And $\alpha\beta = \frac{c}{a} = \frac{p^2}{4}$

Because

$$\alpha^2 + \beta^2 = 1$$

$$\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta = 1$$

Thus, $(\alpha + \beta)^2 - 2\alpha\beta = 1$

$$\left(\frac{-3p}{4}\right)^2 - 2\left(\frac{p^2}{4}\right) = 1$$

$$\frac{9p^2}{16} - \frac{p^2}{2} = 1$$

$$9p^2 - 8p^2 = 16$$

$$\Rightarrow \quad p^2 = 16$$

$$\boxed{p = \pm 4}$$

Q.6.(a) Find x, if 8, x and 18 are in continued proportion. (4)

Ans As x, 8 and 18 are in continued proportion, therefore

$$8 : x :: x : 18$$

Product of means = Product of extremes

$$(x)(x) = (8)(18)$$

$$x^2 = 144$$

$$\sqrt{x^2} = \pm\sqrt{144}$$

$$\boxed{x = \pm 12}$$

(b) Resolve $\frac{1}{3 + x - 2x^2}$ into partial fractions. (4)

Ans For easy solution, we can convert the above function into $\frac{-1}{2x^2 - x - 3}$

Here denominator:

$$D(x) = 2x^2 - x - 3$$

$$= 2x^2 - 3x + 2x - 3$$

$$= x(2x - 3) + 1(2x - 3)$$

$$= (x + 1)(2x - 3)$$

So $$\frac{-1}{2x^2 - x - 3} = \frac{-1}{(x + 1)(2x - 3)}$$

Let $\frac{-1}{(x+1)(2x-3)} = \frac{A}{x+1} + \frac{B}{2x-3}$

By multiplying $(x+1)(2x-3)$ with both sides, (i)

$-1 = A(2x-3) + B(x+1)$

$x + 1 = 0 \Rightarrow x = -1$ in eq. (i)

$-1 = A(2(-1) - 3) + B(-1+1)$

$-1 = A(-2-3) + B(0)$

$-1 = -5A \Rightarrow \boxed{A = \frac{1}{5}}$

Put $2x - 3 = 0 \Rightarrow x = \frac{3}{2}$ in eq. (i)

$-1 = A\left(2\left(\frac{3}{2}\right) - 3\right) + B\left(\frac{3}{2} + 1\right)$

$-1 = A(0) + B\left(\frac{3+2}{2}\right)$

$-1 = \frac{5}{2}B$

$\boxed{\frac{-2}{5} = B}$

So, $\frac{1}{3 + x - 2x^2} = \frac{1}{5(x+1)} - \frac{2}{5(2x-3)}$

Q.7.(a) If $U = \{1, 2, 3, 4, ----, 10\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 4, 5, 8\}$, then prove that $(A - B)' = A' \cup B$ (4)

Ans L.H.S $= (A - B)'$

$= U - (A - B)$

$= \{1, 2, 3, 4, \ldots, 10\} - (\{1, 3, 5, 7, 9\} - \{2, 3, 4, 5, 8\})$

$= \{1, 2, 3, 4, \ldots, 10\} - \{1, 7, 9\}$

$= \{2, 3, 4, 5, 6, 8, 10\}$ (i)

R.H.S $= A' \cup B$

$= (U - A) \cup B$

$= (\{1, 2, 3, 4, \ldots, 10\} - \{1, 3, 5, 7, 9\}) \cup \{2, 3, 4, 5, 8\}$

$= \{2, 4, 6, 8, 10\} \cup \{2, 3, 4, 5, 8\}$

$= \{2, 3, 4, 5, 6, 8, 10\}$ (ii)

From equations (i) and (ii),

L.H.S = R.H.S

(b) Find the standard deviation "S": **(4)**

12, 6, 7, 3, 15, 10, 18, 5

Ans

X	X^2
12	144
6	36
7	49
3	9
15	225
10	100
18	324
5	25
76	912

Formula for Standard Deviation:

$$S.D(x) = S = \sqrt{\left(\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2\right)}$$

$$S = \sqrt{\left(\frac{912}{8}\right) - \left(\frac{76}{8}\right)^2}$$

$$S = \sqrt{114 - 90.25}$$

$$S = \sqrt{23.75}$$

$$\boxed{S = 4.87}$$

Q.8.(a) Prove that: **(4)**

$\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)$

Ans L.H.S $= \cos^4\theta - \sin^4\theta$

$= (\cos^2\theta)^2 - (\sin^2\theta)^2$

$= (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

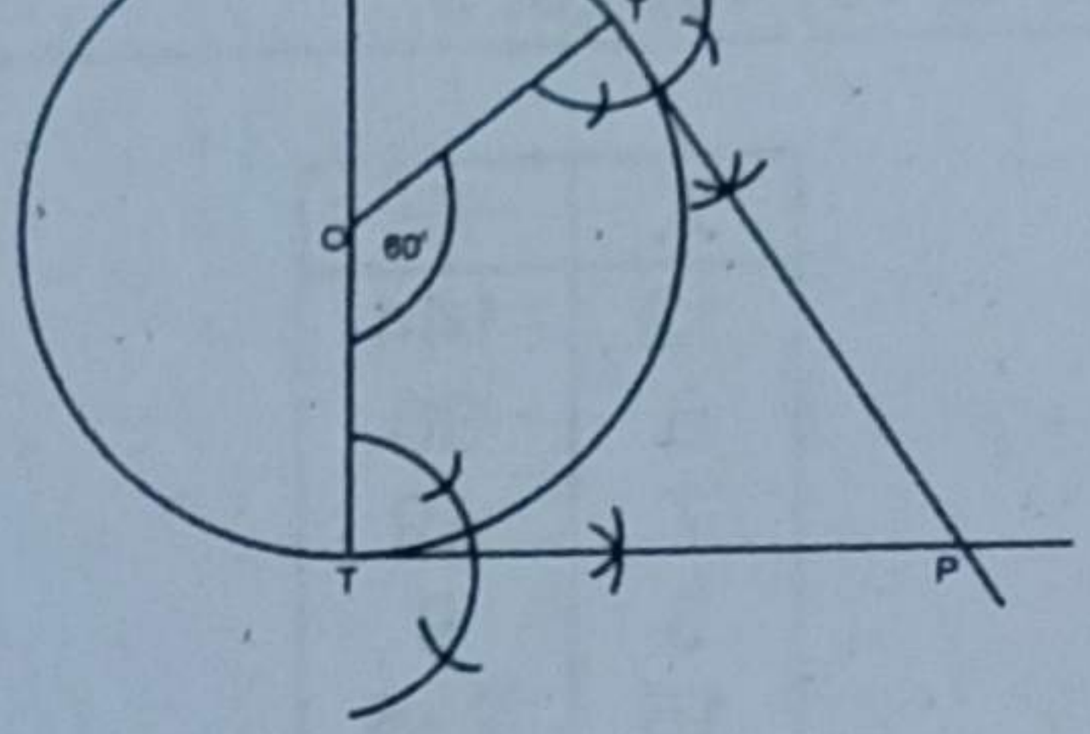
$= 1\,(\cos^2\theta - \sin^2\theta)$

$= \cos^2\theta - \sin^2\theta$

= R.H.S Proved.

(b) Construct a circle of radius 2 cm. Draw two tangents making an angle of 60° with each other.

Ans



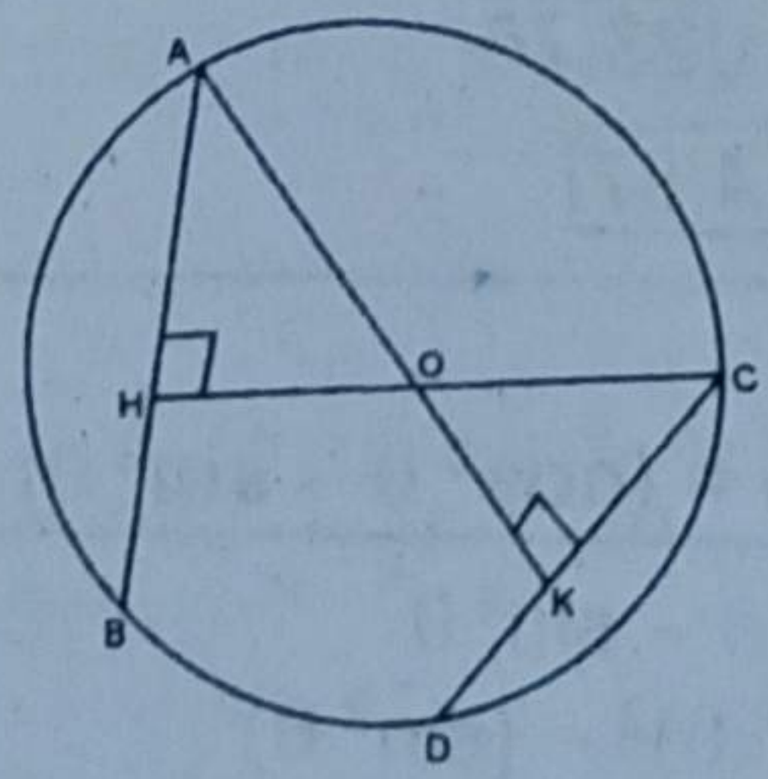
Steps:

1. Draw a circle of 2 cm radius, having O center.
2. Take a diameter $\overline{AOT}$.
3. Make 60° angle on point 'O'.
4. Draw tangents T and T′ with 60° angles, which cut each other at point 'P'.

Here TP and T′P are the required tangents.

Q.9. Prove that two chords of a circle which are equidistant from the centre, are congruent. (4)

Ans



Given:

$\overline{AB}$ and $\overline{CD}$ are two chords of a circle with center at O.
$\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$, so that $m\overline{OH} = m\overline{OK}$.

To Prove:

$m\overline{AB} = m\overline{CD}$

Construction:

Join A and C with O, so that we can form ∠rt Δ^s OAH and OCK.

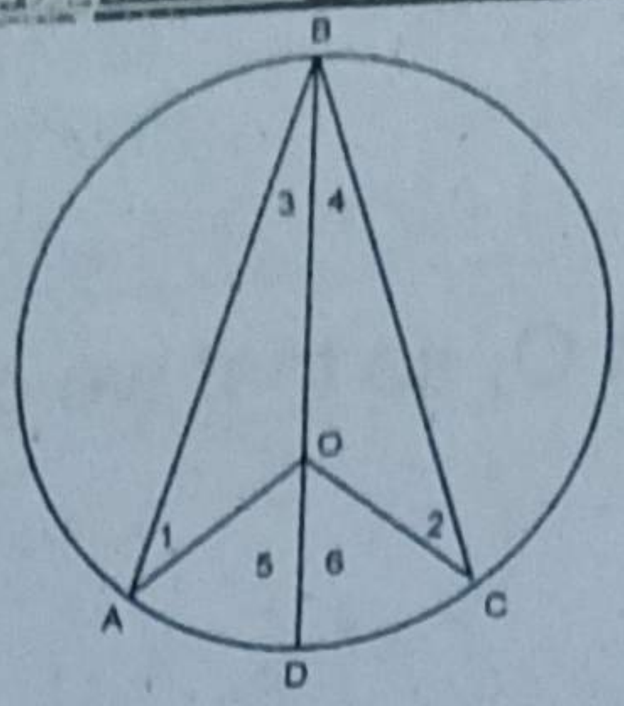
Proof:

Statements	Reasons
In ∠rt Δ^s OAH $\leftrightarrow$ OCK	
$\because$ hyp. $\overline{OA}$ = hyp. $\overline{OC}$	Radii of the same circle
$m\overline{OH} = m\overline{OK}$	Given
$\therefore$ Δ OAH $\cong$ Δ OCK	H.S postulate
So	
$m\overline{AH} = m\overline{CK}$ (i)	Corresponding sides of congruent triangles
But	
$m\overline{AH} = \frac{1}{2} m\overline{AB}$ (ii)	$\overline{OH} \perp$ chord $\overline{AB}$ (Given)
Similarly,	
$m\overline{CK} = \frac{1}{2} m\overline{CD}$ (iii)	$\overline{OK} \perp$ chord $\overline{CD}$ Given
Since $m\overline{AH} = m\overline{CK}$	Already proved in (i)
$\therefore$ $\frac{1}{2} m\overline{AB} = \frac{1}{2} m\overline{CD}$	Using (ii) and (iii)
or	
$m\overline{AB} = m\overline{CD}$	

OR

Prove that the measure of a central of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.

Ans



Given:

$\overset{\frown}{AC}$ is an arc of a circle with center O. Whereas $\angle AOC$ is the central angle and $\angle ABC$ is circumangle.

To Prove:

$m\angle AOC = 2m\angle ABC$

Construction:

Join B with O and produce it to meet the circle at D.

Write angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ as shown in the figure.

Proof:

Statements	**Reasons**
As $m\angle 1 = m\angle 3$ (i)	Angles opposite to equal sides in ΔOAB
and $m\angle 2 = m\angle 4$ (ii)	Angles opposite to equal sides in ΔOBC
Now $m\angle 5 = m\angle 1 + m\angle 3$ (iii)	External angle is the sum of internal opposite angles.
Similarly $m\angle 6 = m\angle 2 + m\angle 4$ (iv)	
Again $m\angle 5 = m\angle 3 + m\angle 3 = 2m\angle 3$ (v)	Using (i) and (iii)
And $m\angle 6 = m\angle 4 + m\angle 4 = 2m\angle 4$ (vi)	Using (ii) and (iv)
Then from fig $\Rightarrow m\angle 5 + m\angle 6 = 2m\angle 3 + 2m\angle 4$	Adding (v) and (vi)
$\Rightarrow m\angle AOC = 2(m\angle 3 + m\angle 4)$ $= 2m\angle ABC$	