

## Inter (Part-II) 2015

**Mathematics****Group-I****PAPER: II****Time: 30 Minutes****(OBJECTIVE TYPE)****Marks: 20**

**Note:** Four possible answers, A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1-  $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 2x}$  is equal to:

(a)  $\frac{2}{3}$       (b)  $\frac{3}{2}$  ✓

(c)  $\frac{1}{6}$       (d)  $\frac{1}{4}$

2- If  $f(x) = x^2 - x$  then  $f(-2)$  is equal to:

(a) 2      (b) 6 ✓  
(c) 0      (d) -6

3-  $\lim_{\delta x \rightarrow 0} \frac{f(x + \delta x) - f(x)}{\delta x}$  is equal to:

(a)  $f(0)$       (b)  $f'(a)$   
(c)  $f'(x)$  ✓      (d)  $f'(0)$

4-  $\frac{d}{dx} x^n$  is equal to:

(a)  $nx^{n-1}$  ✓      (b)  $x^{n-1}$   
(c)  $\frac{x^{n+1}}{n}$       (d)  $nx^{n+1}$

5- If  $f(x) = \cos x$  then  $f'(0)$  is equal to:

(a) 0 ✓      (b) -1  
(c) 1      (d)  $\frac{1}{2}$

6-  $\frac{d}{dx} (\sin h^{-1} x)$  is equal to:

(a)  $\frac{1}{\sqrt{1-x^2}}$       (b)  $\frac{-1}{\sqrt{1-x^2}}$   
(c)  $\frac{1}{\sqrt{1+x^2}}$  ✓      (d)  $\frac{-1}{\sqrt{1+x^2}}$

- 7- If  $f(x) = e^{ax}$  then  $f'(x)$  is equal to:
- (a)  $\frac{e^{ax}}{a}$
  - (b)  $-\frac{e^{ax}}{a}$
  - (c)  $a e^{ax} \checkmark$
  - (d)  $-a e^{ax}$
- 8- The integration is the reverse process of:
- (a) Induction
  - (b) Differentiation  $\checkmark$
  - (c) Tabulation
  - (d) Sublimation
- 9-  $\int \sin x \, dx$  is equal to:
- (a)  $\cos x$
  - (b)  $\sin x$
  - (c)  $-\sin x$
  - (d)  $-\cos x \checkmark$
- 10-  $\int \frac{f'(x)}{f(x)} \, dx$  is equal to:
- (a)  $\ln x$
  - (b)  $\ln f(x) \checkmark$
  - (c)  $\ln f'(x)$
  - (d)  $f(x)$
- 11-  $\int \sec x \tan x \, dx$  is equal to:
- (a)  $\tan x$
  - (b)  $\sec^2 x$
  - (c)  $\tan^2 x$
  - (d)  $\sec x \checkmark$
- 12-  $\int_0^1 x^3 \, dx$  is equal to:
- (a) 4
  - (b) -4
  - (c)  $\frac{1}{4} \checkmark$
  - (d)  $\frac{-1}{4}$
- 13- Solution of differential equation,  $\frac{dy}{dx} = y$  is:
- (a)  $ce^x \checkmark$
  - (b)  $ce^{-x}$
  - (c)  $e^x$
  - (d)  $e^{-x}$
- 14- The point of intersection of medians of a triangle is called:
- (a) Centroid  $\checkmark$
  - (b) Orthocentre
  - (c) Circumcentre
  - (d) Incentre



## Inter (Part-II) 2015

Mathematics

Group-I

PAPER: II

Time: 2.30 Hours

(SUBJECTIVE TYPE)

Marks: 80

## SECTION-I

**2. Write short answers to any EIGHT (8) questions: 16****(i) Show that  $f(x) = x^{2/3} + 6$  is even function.**

**Ans**

$$\begin{aligned} f(x) &= x^{2/3} + 6 \\ &= (x^2)^{1/3} + 6 \\ f(-x) &= \{(-x)^2\}^{1/3} + 6 \\ &= (x^2)^{1/3} + 6 \\ &= x^{2/3} + 6 \\ f(-x) &= f(x) \end{aligned}$$

Therefore  $f(x)$  is even.**(ii) Find the domain and range of  $f^{-1}(x)$  where  $f(x) = 2 + \sqrt{x - 1}$ .**

**Ans**

$$\begin{aligned} f(x) &= 2 + \sqrt{x - 1} \\ f^{-1}(x) &= \frac{1}{f(x)} \\ &= \frac{1}{2 + \sqrt{x - 1}} \end{aligned}$$

 $f(x)$  is not defined when  $x < 1$ Domain of  $f(x) = [1, +\infty)$ Range of  $f(x) = [2, +\infty)$ Domain  $f^{-1} = \text{Range } f = [2, +\infty)$ Range  $f^{-1} = \text{Domain } f = [1, +\infty)$ **(iii) Evaluate  $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$** 

**Ans**

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx} &= \lim_{x \rightarrow a} \frac{ax \frac{\sin ax}{ax}}{bx \frac{\sin bx}{bx}} \\ &= \frac{\lim_{x \rightarrow 0} a \cdot \frac{\sin ax}{ax}}{\lim_{x \rightarrow 0} b \cdot \frac{\sin bx}{bx}} \end{aligned}$$

$$= \frac{a}{b}$$

(iv) Find  $\frac{dy}{dx}$  when  $y = (x - 5)(3 - x)$ .

**Ans**  $y = (x - 5)(3 - x)$

$$\begin{aligned}\frac{dy}{dx} &= (x - 5)(-1) + (3 - x)(1) \\ &= -x + 5 + 3 - x \\ &= 8 - 2x\end{aligned}$$

(v) Compute  $\frac{dy}{dx}$  when  $y = \frac{ax + b}{cx + d}$ .

**Ans**  $y = \frac{ax + b}{cx + d}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(cx + d)(a) - (ax + b)(c)}{(cx + d)^2} \\ &= \frac{acx + ad - acx - bc}{(cx + d)^2} \\ &= \frac{ad - bc}{(cx + d)^2}\end{aligned}$$

(vi) Differentiate  $\sec^{-1} x$  w.r.t.  $x$ .

**Ans**  $y = \sec^{-1} x$

$$x = \sec y$$

$$1 = \sec y \tan y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sec y \cdot \tan y}$$

$$= \frac{1}{\sec y \sqrt{\sec^2 y - 1}}$$

$$\frac{dy}{dx} = \frac{1}{x \sqrt{x^2 - 1}}$$

(vii) If  $y = \tan(p \tan^{-1} x)$  find  $\frac{dy}{dx}$ .

**Ans**  $y = \tan(P \tan^{-1} x)$

$$\frac{dy}{dx} = \sec^2(P \tan^{-1} x) \cdot P \frac{1}{1 + x^2}$$

$$= \{1 + \tan^2(P \tan^{-1} x)\} \frac{P}{1 + x^2}$$

$$= \frac{(1+y^2)P}{1+x^2}$$

(viii) Find  $f'(x)$  when  $f(x) = x^3 e^{1/x}$ .

**Ans**  $f(x) = x^3 \cdot e^{1/x}$

$$\begin{aligned} f'(x) &= x^3 \cdot \left( \frac{-e^{1/x}}{x^2} \right) + e^{1/x} (3x^2) \\ &= e^{1/x} (3x^2 - x) \end{aligned}$$

(ix) Find  $\frac{dy}{dx}$  when  $y = a \cos(\ln x) + b \sin(\ln x)$ .

**Ans**  $y = a \cos(\ln x) + b \sin(\ln x)$

$$\begin{aligned} \frac{dy}{dx} &= a(-\sin \ln x) \cdot \frac{1}{x} + b(\cos \ln x) \cdot \frac{1}{x} \\ &= \frac{1}{x} \{b \cos \ln x - a \sin \ln x\} \end{aligned}$$

(x) Using Maclaurin's series expansion, write first two terms of  $f(x) = \sqrt{1+x}$ .

**Ans**  $f(x) = \sqrt{1+x} = (1+x)^{1/2}$

$$f'(x) = \frac{1}{2}(1+x)^{-1/2}, \quad f''(x) = \frac{-1}{4}(1+x)^{-3/2}$$

$$f(0) = 1, \quad f'(0) = \frac{1}{2}, \quad f''(0) = -\frac{1}{4}$$

Maclaurin's series

$$f(x) = f(0) + f'(0) \cdot x + \frac{x^2}{2!} f(0) + \dots$$

$$\sqrt{1+x} = 1 + x \cdot \frac{1}{2} + \frac{x^2}{2!} \left(-\frac{1}{4}\right) + \dots$$

$$= 1 + \frac{x}{2} + \dots$$

(xi) Find  $\frac{dy}{dx}$  when  $y = \sin h^{-1} \left(\frac{x}{2}\right)$ .

**Ans**  $y = \sin h^{-1} \frac{x}{2}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+\frac{x^2}{4}}} \cdot \frac{1}{2}$$

$$= \frac{1}{\sqrt{4 + x^2}}$$

(xii) Find critical values of  $f(x) = \sin x + \cos x$ .

**Ans**  $f(x) = \sin x + \cos x$

$$f'(x) = \cos x - \sin x$$

For critical values

$$f'(x) = 0$$

$$\cos x - \sin x = 0 \Rightarrow \cos x = \sin x$$

$$\cos x = \sin x \text{ is satisfied when } x = \frac{\pi}{4}, \frac{\pi}{4} \text{ for } x \in [0, 2\pi]$$

### 3. Write short answers to any EIGHT (8) questions: 16

(i) Using differential, find  $\frac{dy}{dx}$  when  $xy - \ln x = c$ .

**Ans**  $xy - \ln x = c$

$$x dy + y dx - \frac{1}{x} dx = 0$$

$$x dy = \left(\frac{1}{x} - y\right) dx$$

$$\frac{dy}{dx} = \frac{1 - xy}{x^2}$$

(ii) Evaluate  $\int \frac{dx}{\sqrt{x+1} - \sqrt{x}}$ .

**Ans**  $\int \frac{dx}{\sqrt{x+1} - \sqrt{x}}$

$$= \int \frac{(\sqrt{x+1} + \sqrt{x}) dx}{(\sqrt{x+1} - \sqrt{x})(\sqrt{x+1} + \sqrt{x})}$$

$$= \int \frac{\sqrt{x+1} + \sqrt{x}}{x+1-x} dx$$

$$= \int \sqrt{x+1} dx + \int \sqrt{x} dx$$

$$= \frac{2}{3}(x+1)^{3/2} + \frac{2}{3}(x)^{3/2} + c$$

(iii) Evaluate  $\int \sqrt{1 - \cos 2x} dx$  ( $1 - \cos 2x > 0$ ).

**Ans**  $\int \sqrt{1 - \cos 2x} . dx$

$$= \int \sqrt{2 \sin^2 x} dx$$

$$= \sqrt{2} \int \sin x \, dx$$

$$= -\sqrt{2} \cos x + C$$

(iv) Find  $\int \frac{dx}{x (\ln 2x)^3}$   $x > 0$ .

**Ans**  $\int \frac{dx}{x (\ln 2x)^3} \quad x > 0$

$$= \int \frac{1}{(\ln 2x)^3} \cdot \frac{1}{x} dx$$

$$\ln 2x = t$$

$$\frac{1}{2x} \cdot 2 \cdot dx = dt$$

$$\frac{1}{x} dx = dt$$

$$\int \frac{1}{(\ln 2x)^3} \cdot \frac{1}{x} dx = \int \frac{1}{t^3} \cdot dt$$

$$\int t^{-3} dt = \frac{t^{-2}}{-2} + C$$

$$= -\frac{1}{2t^2} + C$$

$$= -\frac{1}{2(\ln 2x)^2} + C$$

(v) Evaluate  $\int x^5 \ln x \, dx$ .

**Ans**  $\int x^5 \ln x \, dx$

$$= \ln x \cdot \frac{x^6}{6} - \int \frac{x^6}{6} \cdot \frac{1}{x} dx$$

$$= \frac{x^6}{6} \ln x - \frac{1}{6} \int x^5 dx$$

$$= \frac{x^6}{6} \ln x - \frac{1}{36} x^6 + C$$

(vi) Evaluate  $\int \frac{2a}{a^2 - x^2} dx$   $x < a$ .

**Ans**  $\int \frac{2a}{a^2 - x^2} dx$

$$= \int \frac{(a+a+x-x)}{(a+x)(a-x)} dx$$

$$\begin{aligned}
 &= \int \frac{1}{a-x} dx + \int \frac{1}{a+x} dx \\
 &= -\ln(a-x) + \ln(a+x) + C \\
 &= \ln \frac{a+x}{a-x} + C
 \end{aligned}$$

(vii) Evaluate  $\int_{-1}^2 [x + |x|] dx$ .

**Ans**  $\int_{-1}^2 (x + |x|) dx$

$$\begin{aligned}
 &= \int_{-1}^0 (x + |x|) dx + \int_0^2 (x + |x|) dx \\
 &= \int_{-1}^0 [x + (-x)] dx + \int_0^2 [x + (x)] dx \\
 &= \int_{-1}^0 0 dx + \int_0^2 2x dx \\
 &= 0 + |x^2|_0^2 = 4
 \end{aligned}$$

(viii) Evaluate  $\int_0^3 \frac{dx}{x^2 + 9}$ .

**Ans**  $\int_0^3 \frac{dx}{x^2 + 9}$

$$\begin{aligned}
 &= \frac{1}{3} \left[ \tan^{-1} \frac{x}{3} \right]_0^3 \\
 &= \frac{1}{3} [\tan^{-1} 1 - \tan^{-1} 0] \\
 &= \frac{\pi}{12}
 \end{aligned}$$

(ix) Find the area bounded by  $y = \cos x$  from  $x = -\frac{\pi}{2}$  to  $x = \frac{\pi}{2}$ .

**Ans**  $A = \int_{-\pi/2}^{\pi/2} y dx = \int_{-\pi/2}^{\pi/2} \cos x dx$

$$\begin{aligned}
 &= |\sin x|_{-\pi/2}^{\pi/2}
 \end{aligned}$$

$$\left[ \sin \frac{\pi}{2} - \sin \left( -\frac{\pi}{2} \right) \right] = 2$$

(x) Solve the differential equation  $x^2 (2y + 1) \frac{dy}{dx} - 1 = 0$ .

**Ans**  $x^2 (2y + 1) \frac{dy}{dx} - 1 = 0$

$$(2y + 1) dy = \frac{1}{x^2} dx$$

$$\int (2y + 1) dy = \int x^{-2} dx$$

$$y^2 + y = c - \frac{1}{x}$$

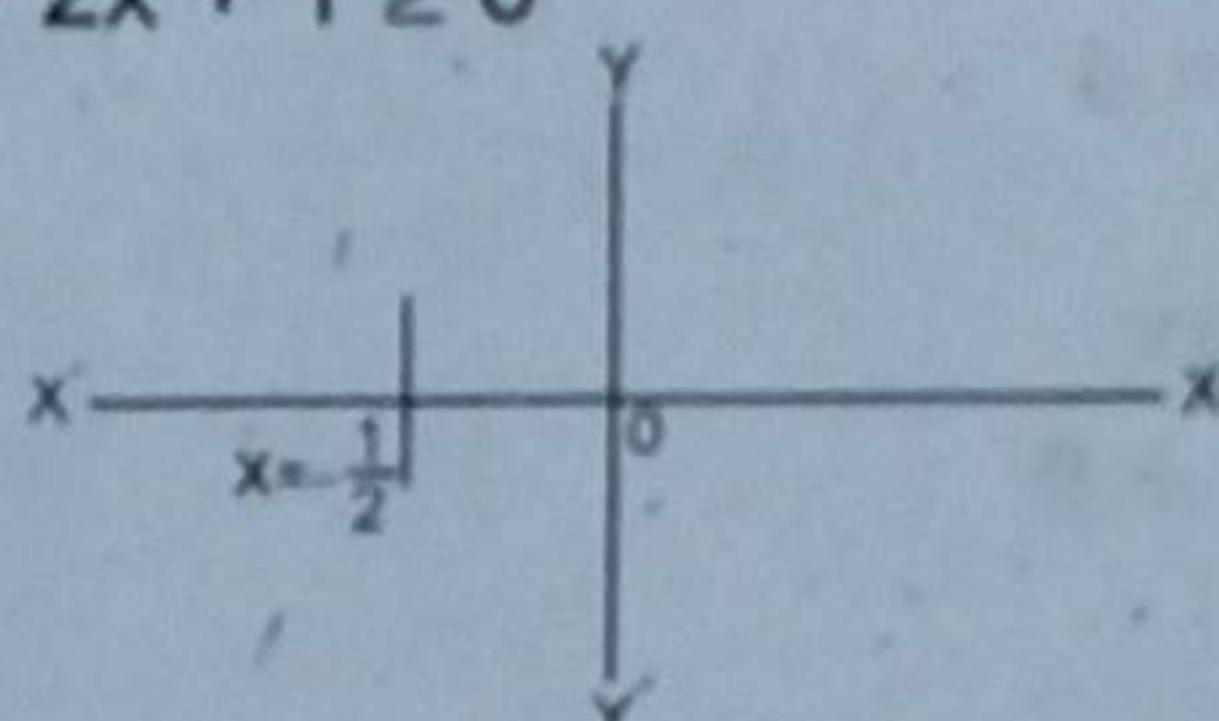
(xi) Graph the solution region of  $2x + 1 \geq 0$ .

**Ans**  $2x + 1 \geq 0$   
Associated equation

$$2x + 1 = 0$$

$$x = -\frac{1}{2}$$

$(0, 0)$  satisfy  $2x + 1 \geq 0$



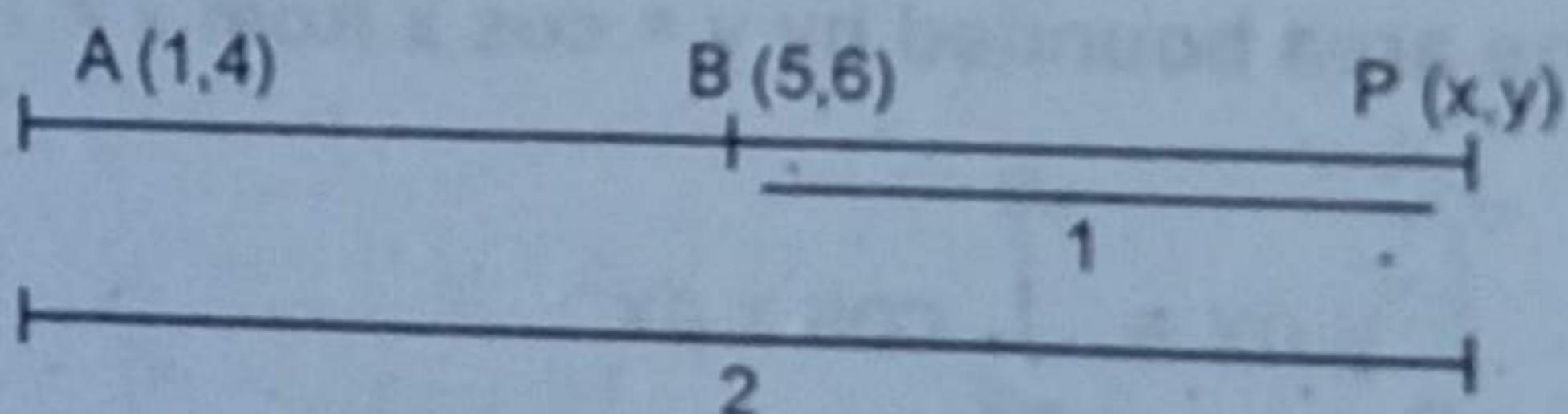
(xii) What are problem constraints?

**Ans** The system of linear inequalities involved in the problem concerned are called problem constraints.

**4. Write short answers to any NINE (9) questions:** 18

(i) Find the point P on the join of A (1, 4) and B (5, 6) that is twice as far from A as B is from A and lies on the same side of A as B does.

**Ans**



$$x = \frac{m_2 x_1 - m_1 x_2}{m_2 - m_1}$$

$$x = \frac{1 \times 1 - 2 \times 5}{1 - 2} = \frac{1 - 10}{-1} = \frac{-9}{-1} = 9$$

$$y = \frac{1 \times 4 - 2 \times 6}{1 - 2} = \frac{4 - 12}{-1} = \frac{-8}{-1} = 8$$

P(9, 8)

B is the mid-point

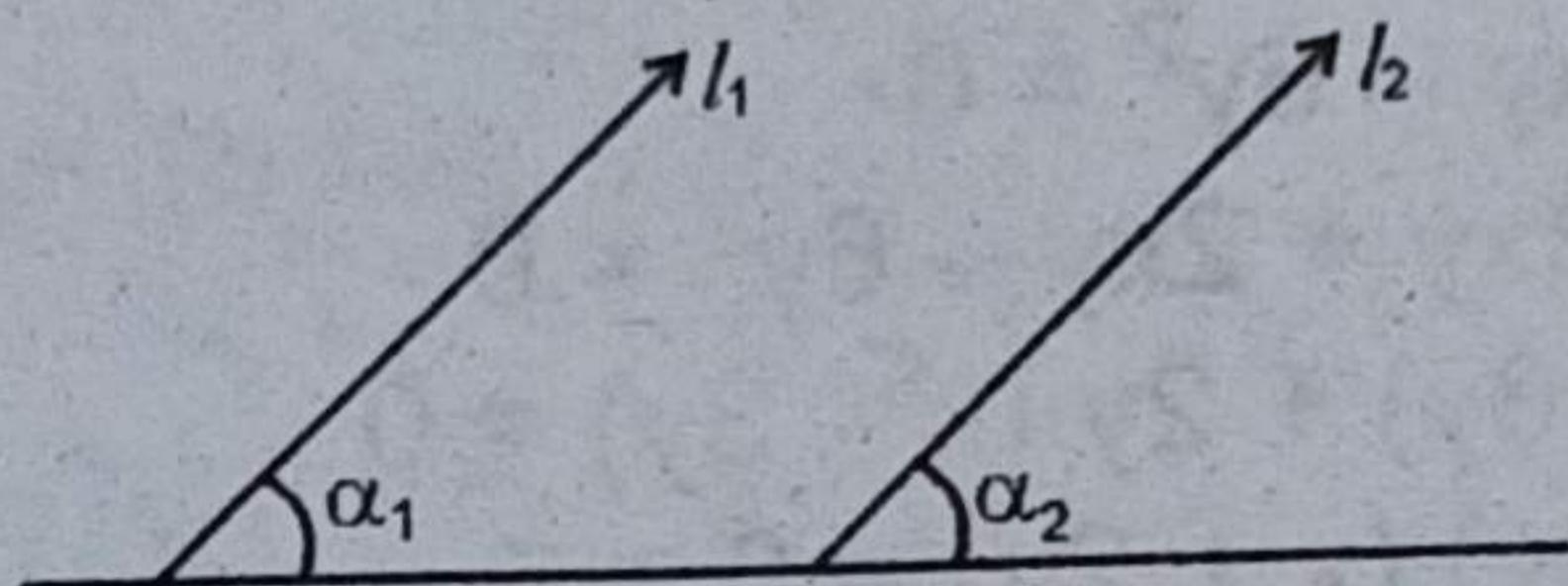
$$\therefore 5 = \frac{1+x}{2} \Rightarrow x = 9$$

$$6 = \frac{4+y}{2} \Rightarrow y = 12 - 4 = 8$$

P(9, 8)

- (ii) Two lines  $l_1$  and  $l_2$  with respective slopes  $m_1$  and  $m_2$  are parallel iff  $m_1 = m_2$ .

**Ans**



Let  $l_1$  is || to  $l_2$

$$\therefore \alpha_1 = \alpha_2$$

$$\Rightarrow \tan \alpha_1 = \tan \alpha_2$$

$$\Rightarrow m_1 = m_2$$

Conversely

$$\text{Let } m_1 = m_2$$

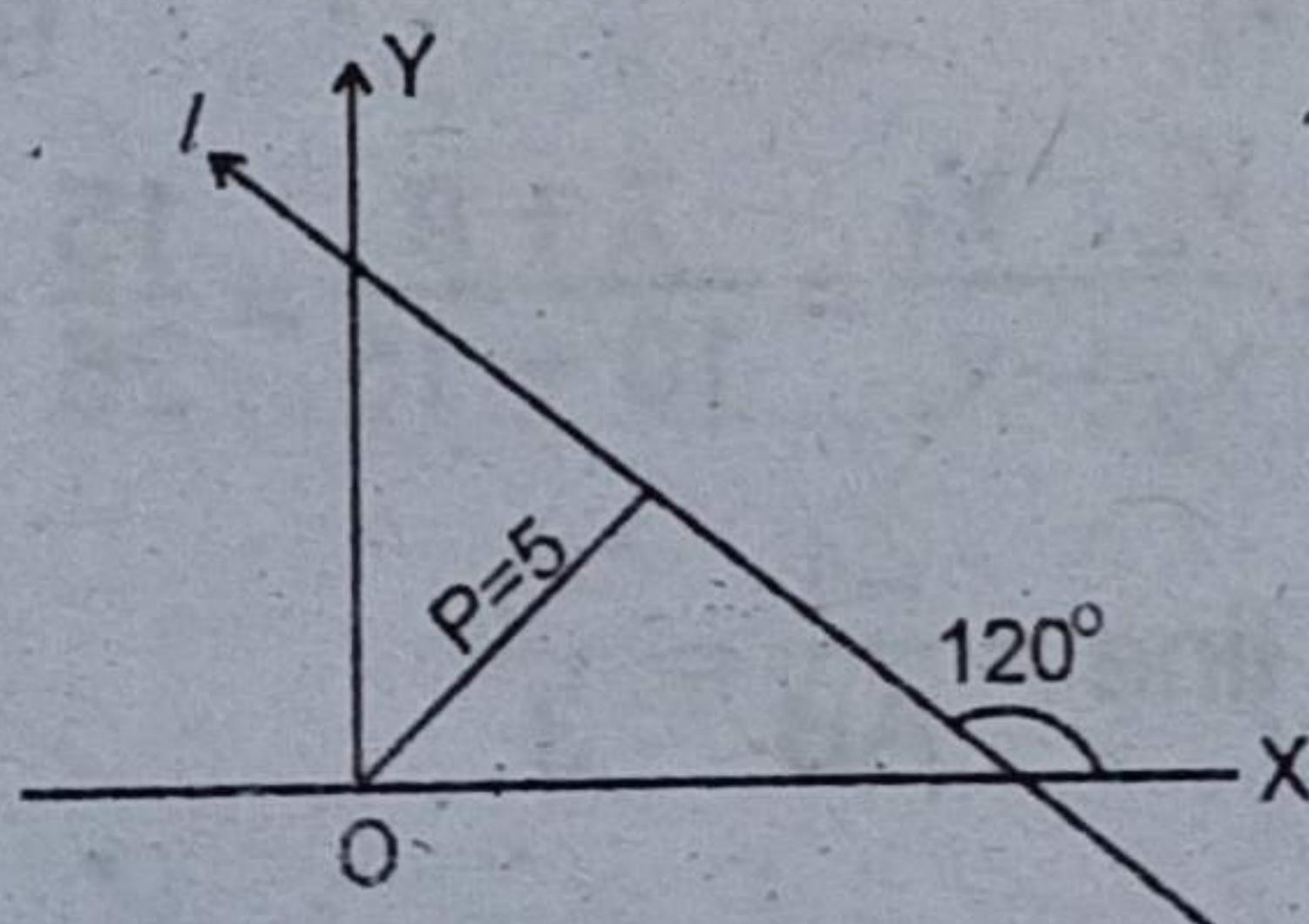
$$\Rightarrow \tan \alpha_1 = \tan \alpha_2$$

$$\Rightarrow \alpha_1 = \alpha_2$$

$$\Rightarrow l_1 // l_2$$

- (iii) If length of perpendicular from origin to a line is 5 units and its inclination is  $120^\circ$ , find slope and y-intercept of the line.

**Ans**



Equation of line is

$$x \cos \alpha + y \sin \alpha = P$$

$$x \cos 120^\circ + y \sin 120^\circ = 5$$

$$x \cdot \left(-\frac{1}{2}\right) + y \cdot \frac{\sqrt{3}}{2} = 5$$

$$-\frac{1}{2}x + \frac{\sqrt{3}}{2}y = 5$$

$$\sqrt{3}y = x + 10$$

$$y = \frac{1}{\sqrt{3}}x + \frac{10}{\sqrt{3}} \quad (y = mx + c)$$

$$\text{So slope } = \frac{1}{\sqrt{3}}, \quad \text{c } = \text{y-intercept } = \frac{10}{\sqrt{3}}$$

- (iv) Find the lines represented by  $x^2 - xy - 6y^2 = 0$ , also find the angle between them.

**Ans**

$$x^2 - xy - 6y^2 = 0$$

$$x^2 - 3xy + 2xy - 6y^2 = 0$$

$$x(x - 3y) + 2y(x - 3y) = 0$$

$$(x - 3y)(x + 2y) = 0$$

$x - 3y = 0, x + 2y = 0$  are required lines.

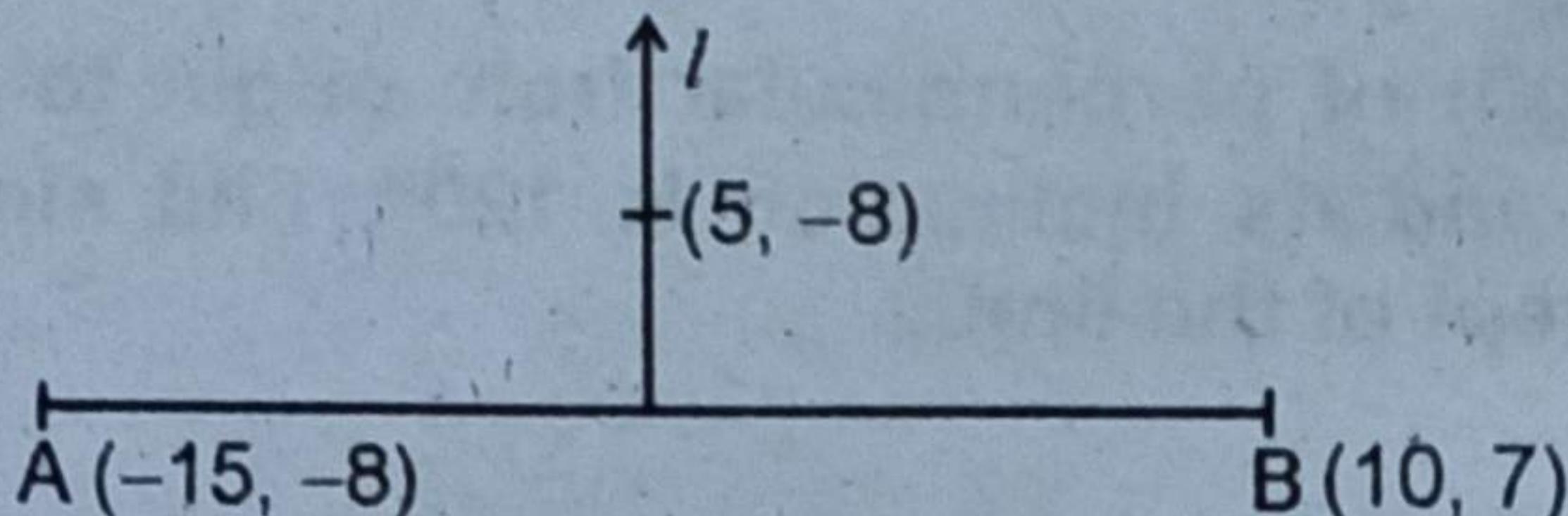
$$\tan \theta = \frac{2\sqrt{h^2 - ab}}{a + b} \quad a = 1, b = -6, 2h = -1, h = -\frac{1}{2}$$

$$= \frac{2\sqrt{\frac{1}{4} - 1 \times -6}}{1 - 6} = \frac{2\sqrt{\frac{1}{4} + 6}}{-5} = \frac{2\sqrt{\frac{25}{4}}}{-5} = \frac{2 \cdot \frac{5}{2}}{-5} = -1$$

$$\theta = 135^\circ$$

- (v) Find an equation of the line through  $(5, -8)$  and perpendicular to the join of  $A(-15, -8)$ ,  $B(10, 7)$ .

**Ans**



$$\text{Slope of AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-8)}{10 - (-15)} = \frac{15}{25} = \frac{3}{5}$$

$$\text{Slope of Req. line} = \frac{-1}{AB} = \frac{-5}{3}$$

$\therefore$  Eq. of line passing through  $(5, -8)$  and having slope  $\frac{-5}{3}$  is  $\infty$ .

$$y - y_1 = m(x - x_1)$$

$$y + 8 = \frac{-5}{3}(x - 5)$$

$$3y + 24 = -5x + 25$$

$$5x + 3y - 1 = 0$$

- (vi) Find the coordinates of the points of intersection of the line  $2x + y = 5$  and  $x^2 + y^2 + 2x - 9 = 0$ .

**Ans**  $2x + y = 5 \quad x^2 + y^2 + 2x - 9 = 0$

$$y = 5 - 2x$$

$$\therefore x^2 + (5 - 2x)^2 + 2x - 9 = 0$$

$$x^2 + 25 + 4x^2 - 20x + 2x - 9 = 0$$

$$5x^2 - 18x + 16 = 0$$

$$5x^2 - 10x - 8x + 16 = 0$$

$$5x(x - 2) - 8(x - 2) = 0$$

$$(x - 2)(5x - 8) = 0$$

$$x = 2, x = \frac{8}{5}$$

$$y = 5 - 2x$$

$$y = 5 - 4 = 1$$

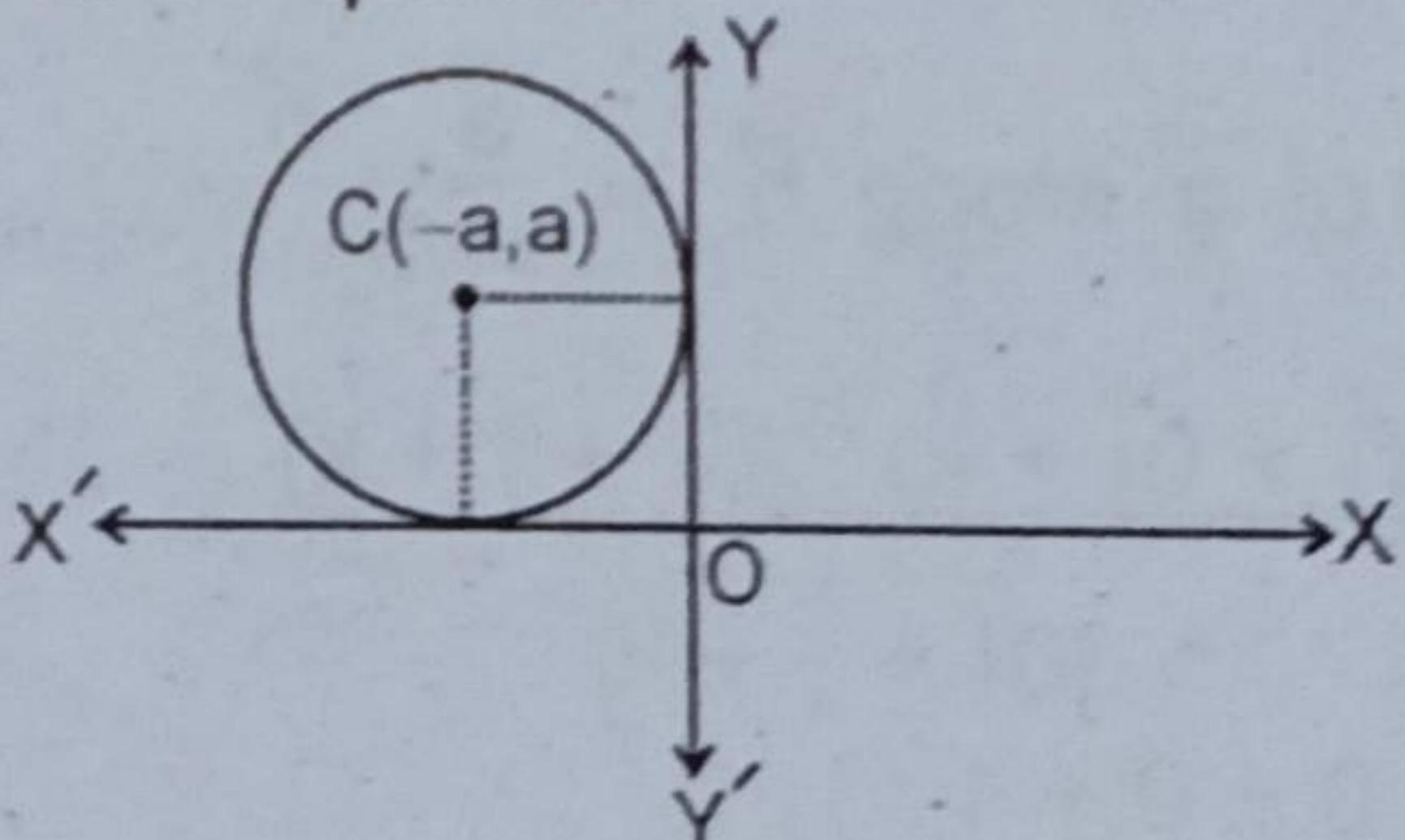
$$y = 5 - \frac{16}{5} = \frac{25 - 16}{5} = \frac{9}{5}$$

$\therefore$  Points of Intersections are

$$(2, 1), (\frac{8}{5}, \frac{9}{5})$$

- (vii) Find an equation of a circle of radius 'a' and lying in the second quadrant such that it is tangent to both the axes.

**Ans** Circle lies in 2<sup>nd</sup> quadrant



$\therefore$  Centre of circle is  $(-a, a)$

and Radius is 'a'.

So equation circle is  $(x + a)^2 + (y - a)^2 = a^2$ .

**(viii) Find the focus and vertex of parabola  $y^2 = -8(x - 3)$ .**

**Ans**

$$y^2 = -8(x - 3)$$

$$\text{Let } Y = y, X = x - 3$$

$$\therefore Y^2 = -8X$$

Vertex is  $V(0, 0)$

$$x = 0, y = 0$$

$$x - 3 = 0, y = 0$$

$$x = 3, y = 0$$

$$x = 3, y = 0$$

$$\text{vertex} = (3, 0)$$

$$4a = -8$$

$$a = -2$$

$$\text{Focus} = F(-a, 0)$$

$$X = -a, Y = 0$$

$$X = -2, Y = 0$$

$$x - 3 = -2$$

$$x = 1$$

$$Y = y = 0$$

Focus is  $(1, 0)$ .

**(ix) Find equation of hyperbola with foci  $(\pm 5, 0)$  and vertex  $(3, 0)$ .**

**Ans**

$$\text{Foci } (\pm 5, 0) \quad \text{vertex } (3, 0)$$

$$\text{Here } c = 5, a = 3, b = ?$$

$$b^2 = c^2 - a^2 = (5)^2 - (3)^2 = 25 - 9 = 16$$

$$b = 4$$

$$\text{Equation of hyperbola is } \frac{x^2}{(3)^2} - \frac{y^2}{(4)^2} = 1$$

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

**(x)**

**Calculate projection of  $\underline{a}$  along  $\underline{b}$  when  $\underline{a} = \underline{i} + \underline{k}$ ,  $\underline{b} = \underline{j} + \underline{k}$ .**

**Ans**

$$\text{Projection of } \vec{a} \text{ along } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$= \frac{(\hat{i} + 0\hat{j} + \hat{k}) \cdot (0\hat{i} + \hat{j} + \hat{k})}{|0\hat{i} + \hat{j} + \hat{k}|}$$

$$= \frac{0 + 0 + 1}{\sqrt{0 + 1 + 1}} = \frac{1}{\sqrt{2}}$$

(xi) Find the angle between the vectors  $\underline{u} = 2\hat{i} - \hat{j} + \hat{k}$  and  $\underline{v} = -\hat{i} + \hat{j}$ .

**Ans**  $\vec{u} = 2\hat{i} - \hat{j} + \hat{k}, \vec{v} = -\hat{i} + \hat{j} + 0\hat{k}$

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{(\vec{u})(\vec{v})} = \frac{(2\hat{i} - \hat{j} + \hat{k}) \cdot (-\hat{i} + \hat{j} + 0\hat{k})}{|2\hat{i} - \hat{j} + \hat{k}| |-\hat{i} + \hat{j} + 0\hat{k}|}$$

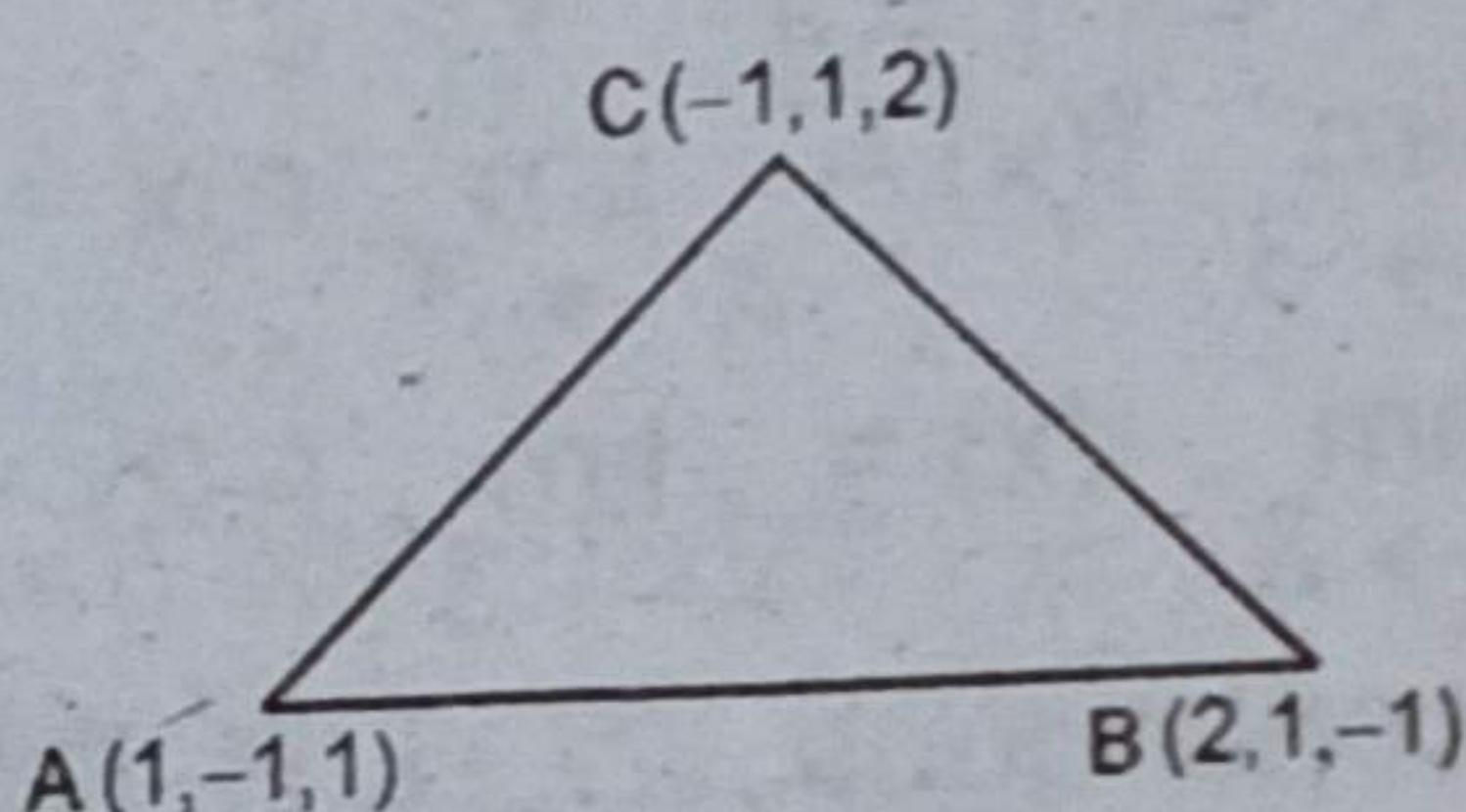
$$= \frac{-2 - 1 + 0}{\sqrt{4 + 1 + 1} \cdot \sqrt{1 + 1 + 0}} = \frac{-3}{\sqrt{6} \times \sqrt{2}}$$

$$\cos \theta = \frac{-\sqrt{3} \times \sqrt{3}}{\sqrt{3} \times \sqrt{2} \times \sqrt{2}} = -\frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = 150^\circ$$

(xii) Find the area of the triangle with vertices A (1, -1, 1), B (2, 1, -1) C (-1, 1, 2).

**Ans**



$$\vec{AB} = (2 - 1)\hat{i} + (1 + 1)\hat{j} + (-1 - 1)\hat{k} = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{AC} = (-1 - 1)\hat{i} + (1 + 1)\hat{j} + (2 - 1)\hat{k} = -2\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -2 \\ -2 & 2 & 1 \end{vmatrix} = 6\hat{i} + 3\hat{j} + 6\hat{k}$$

$$|\vec{AB} \times \vec{AC}| = \sqrt{36 + 9 + 36} = \sqrt{81} = 9$$

$$\text{Area of } \triangle ABC = \frac{1}{2} |\vec{AB} \times \vec{AC}| = \frac{9}{2}$$

(xiii) Find the direction cosines for  $\vec{PQ}$ , where P (2, 1, 5), Q (1, 3, 1).

**Ans** P(2, 1, 5), Q(1, 3, 1)

$$\vec{PQ} = (1 - 2)\hat{i} + (3 - 1)\hat{j} + (1 - 5)\hat{k}$$

$$= -\hat{i} + 2\hat{j} - 4\hat{k}$$

$$|\vec{PQ}| = \sqrt{1 + 4 + 16} = \sqrt{21}$$

Direction cosines are

$$\left( \frac{-1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{-4}{\sqrt{21}} \right)$$

## SECTION-II

**NOTE: Attempt any Three (3) questions.**

**Q.5.(a) Find the values m and n, so that given function  $f(x)$  is continuous** (5)

$$f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$$

**Ans**

$$f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$$

$$\text{L.H. lim} = \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} mx = 3m$$

$$\text{R.H. lim} = \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (-2x + 9) = -6 + 9 = 3$$

Since  $f(x)$  is continuous  $\therefore \text{L.H. lim} = \text{R.H. lim}$   
 $3m = 3$

$$\boxed{m = 1}$$

$$\text{Also } f(3) = n$$

$$\text{Also } \lim_{x \rightarrow 3^+} f(x) = 3$$

$$\text{Since } \lim_{x \rightarrow 3^+} f(x) = f(3)$$

$$\boxed{3 = n}$$

**(b) Differentiate  $\cos \sqrt{x}$  from the first principles.**

(5)

**Ans**

$$y = \cos \sqrt{x}$$

$$y + \delta y = \cos \sqrt{x + \delta x}$$

$$\delta y = \cos \sqrt{x + \delta x} - \cos \sqrt{x}$$

$$\delta y = -2 \sin \frac{\sqrt{x + \delta x} + \sqrt{x}}{2} \cdot \sin \frac{\sqrt{x + \delta x} - \sqrt{x}}{2}$$

$$\frac{\delta y}{\delta x} = \frac{-2 \sin \frac{\sqrt{x + \delta x} + \sqrt{x}}{2} \cdot \sin \frac{\sqrt{x + \delta x} - \sqrt{x}}{2}}{\delta x}$$

$$\frac{\delta y}{\delta x} = \frac{-2 \sin \frac{\sqrt{x + \delta x} + \sqrt{x}}{2}}{(\sqrt{x + \delta x} + \sqrt{x})} \cdot \frac{\sin \frac{\sqrt{x + \delta x} - \sqrt{x}}{2}}{(\sqrt{x + \delta x} - \sqrt{x})}$$

$$\frac{\delta y}{\delta x} = \left( \frac{-\sin \frac{\sqrt{x + \delta x} + \sqrt{x}}{2}}{\sqrt{x + \delta x} + \sqrt{x}} \right) \times \left( \frac{\sin \frac{\sqrt{x + \delta x} - \sqrt{x}}{2}}{\frac{\sqrt{x + \delta x} - \sqrt{x}}{2}} \right)$$

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \frac{-\sin \frac{2\sqrt{x}}{2}}{2\sqrt{x}} \times 1$$

$$\frac{dy}{dx} = \frac{-\sin \sqrt{x}}{2\sqrt{x}}$$

**Q.6.(a) Show that**  $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln(x + \sqrt{x^2 - a^2}) + c.$  (5)

**Ans**  $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln(x + \sqrt{x^2 - a^2}) + c$

Put  $x = a \sec \theta$

$dx = a \sec \theta \tan \theta d\theta$

$$\begin{aligned}\therefore \int \frac{dx}{\sqrt{x^2 - a^2}} &= \int \frac{a \sec \theta \tan \theta d\theta}{\sqrt{a^2 \sec^2 \theta - a^2}} \\ &= \int \frac{a \sec \theta \tan \theta d\theta}{a \sqrt{\tan^2 \theta}} = \int \sec \theta d\theta \\ &= \ln(\sec \theta + \tan \theta) + c_1\end{aligned}$$

since  $\sec \theta = \frac{x}{a} \quad \therefore \tan \theta \sqrt{\sec^2 \theta - 1} = \sqrt{\frac{x^2}{a^2} - 1} = \frac{\sqrt{x^2 - a^2}}{a^2}$

$$\begin{aligned}\therefore \int \frac{dx}{\sqrt{x^2 - a^2}} &= \ln \left( \frac{x}{a} + \frac{\sqrt{x^2 - a^2}}{a} \right) \\ &= \ln(x + \sqrt{x^2 - a^2}) - \ln a + c_1\end{aligned}$$

Let,  $-\ln a + c_1 = c$

So  $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln(x + \sqrt{x^2 - a^2}) + c$

- (b) Find equations of two parallel lines perpendicular to  $2x - y + 3 = 0$  such that the product of the x-intercept and y-intercept of each is 3. (5)

**Ans** Let the required eq. of two || lines  $\perp$  to  $2x - y + 3 = 0$

$$\text{is } x + 2y + k = 0 \quad \dots(A)$$

For x-Intercept put  $y = 0$  in (A)  $\therefore x = -k$

$$\text{For y-Intercept put } x = 0 \text{ in (A)} \quad y = \frac{-k}{2}$$

Product of (x-Intercept) (y-Intercept) = 3

$$(-k) \left(\frac{-k}{2}\right) = 3$$

$$k^2 = 6$$

$$k = \pm \sqrt{6}$$

$\therefore$  Eq (A) becomes  $x + 2y \pm \sqrt{6} = 0$

$$\text{i.e., } x + 2y + \sqrt{6} = 0$$

$$x + 2y - \sqrt{6} = 0$$

- Q.7.(a) Evaluate  $\int_0^{\pi/4} \frac{\sin x - 1}{\cos^2 x} dx$ . (5)

**Ans** Evaluate  $\int_0^{\pi/4} \frac{\sin x - 1}{\cos^2 x} dx$

$$= \int_0^{\pi/4} \left[ \frac{\sin x}{\cos^2 x} - \frac{1}{\cos^2 x} \right] dx$$

$$= \int_0^{\pi/4} [\tan x \cdot \sec x - \sec^2 x] dx$$

$$= |\sec x - \tan x|_0^{\pi/4}$$

$$= (\sec \frac{\pi}{4} - \tan \frac{\pi}{4}) - (\sec 0 - \tan 0)$$

$$= \sqrt{2} - 1 - (1 - 0)$$

$$= \sqrt{2} - 2$$

- (b) Graph the feasible region and also find the corner points: (5)

$$2x - 3y \leq 6, 2x + 3y \leq 12, x \geq 0, y \geq 0$$

**Ans**  $2x - 3y \leq 6$

$2x + 3y \leq 12 \quad x \geq 0, y \geq 0$

Associated eqs. are:

$2x - 3y = 6$

(i)

$$2x + 3y = 12$$

From eq. (ii)

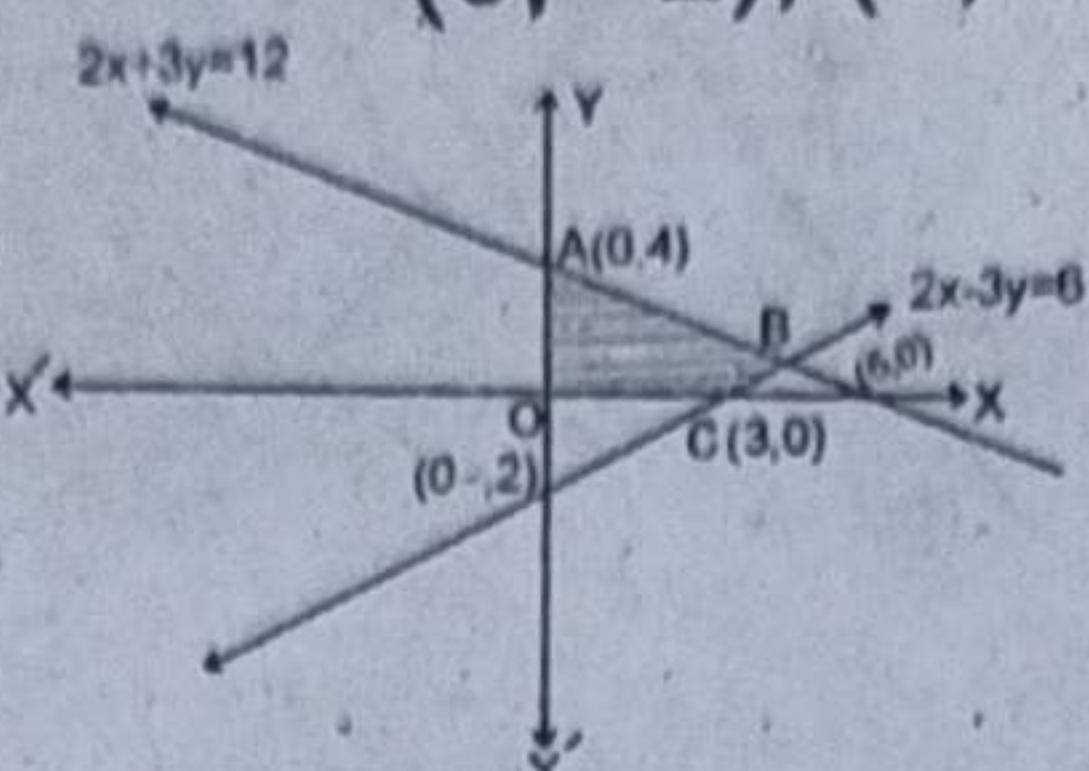
x	0	6
y	4	0

(0, 4), (6, 0)

From eq. (i)

x	0	3
y	-2	0

(0, -2), (3, 0)



By solving

$$2x - 3y = 6$$

$$\pm 2x \pm 3y = \pm 12$$

$$-6y = -6$$

$$y = 1$$

$$B\left(\frac{9}{2}, 1\right)$$

$$2x - 3y = 6$$

$$2x + 3y = 12$$

$$4x = 18$$

$$x = \frac{9}{2}$$

Feasible region is OABC

Corner pts. are (0, 0), C(3, 0)

$$A(0, 4), B\left(\frac{9}{2}, 1\right)$$

**Q.8.(a) Show that  $2x + 3y - 13 = 0$  is tangent to circle  $x^2 + y^2 + 6x - 4y = 0$ . (5)**

**Ans**

$$x^2 + y^2 + 6x - 4y = 0$$

$$2g = 6,$$

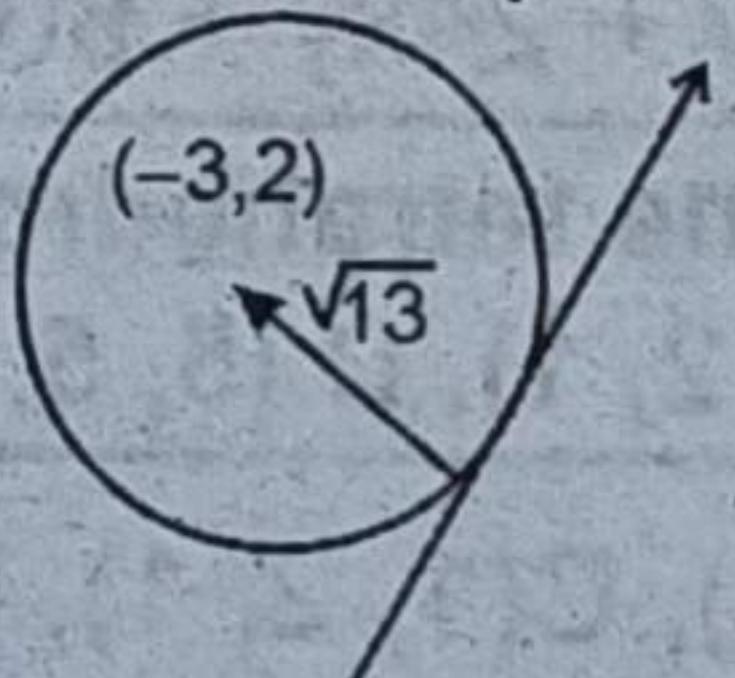
$$2f = -4$$

$$g = 3$$

$$f = -2$$

$$\text{centre } (-g, -f) = (-3, 2)$$

$$\text{radius } r = \sqrt{g^2 + f^2 - c} = \sqrt{9 + 4} = \sqrt{13}$$



$$2x + 3y - 13 = 0$$

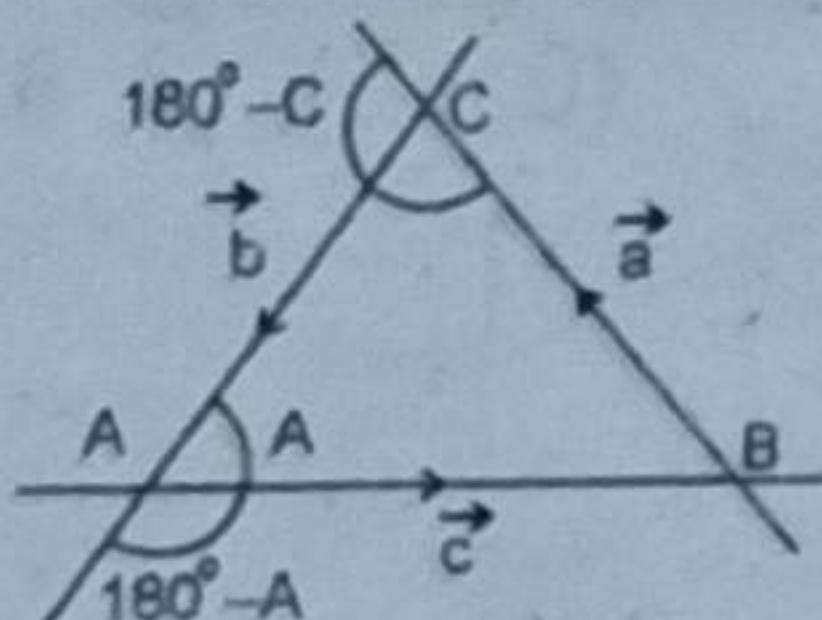
$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$d = \frac{|2(-3) + 3(2) - 13|}{\sqrt{(2)^2 + (3)^2}} = \frac{|-13|}{\sqrt{13}} = \frac{13}{\sqrt{13}} = \sqrt{13}$$

$\therefore$  line  $2x + 3y - 13 = 0$  is tangent to the circle.

(b) Using vector prove that  $b = c \cos A + a \cos C$ . (5)

**Ans**



$$\vec{a} + \vec{b} + \vec{c} = 0$$

$$\vec{b} = -(\vec{a} + \vec{c})$$

Taking cross product with  $\vec{b}'$

$$\vec{b} \cdot \vec{b} = -(\vec{a} + \vec{c}) \cdot \vec{b}$$

$$b^2 = -\vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{b}$$

$$b^2 = -ab \cos(180^\circ - C) - cb \cos(180^\circ - A)$$

$$b = a \cos C + c \cos A$$

Q.9.(a) Find an equation of the parabola whose focus is F (-3, 4), and directrix line is  $3x - 4y + 5 = 0$ . (5)

**Ans** F(-3, 4) eq of directrix is  $3x - 4y + 5 = 0$

Let P(x, y) be any pt. on the parabola

$$|PF| = |PM|$$

$$\therefore \sqrt{(x + 3)^2 + (y - 4)^2} = \frac{|3x - 4y + 5|}{\sqrt{(3)^2 + (4)^2}}$$

$$(x + 3)^2 + (y - 4)^2 = \frac{|3x - 4y + 5|^2}{9 + 16}$$

$$x^2 + 6x + 9 + y^2 - 8y + 16 = \frac{9x^2 + 16y^2 + 25 - 24xy + 30x - 40y}{25}$$

$$25x^2 + 25y^2 + 150x - 200y + 625 = 9x^2 + 16y^2 - 24xy + 30x - 40y + 25$$

$$16x^2 + 9y^2 + 120x - 160y + 24xy + 600 = 0$$

(b) Find the volume of the tetrahedron with the vertices A (0, 1, 2), B (3, 2, 1) C (1, 2, 1), D (5, 5, 6). (5)

**Ans**

A(0, 1, 2), B(3, 2, 1), C(1, 2, 1), D(5, 5, 6)

$$\vec{AB} = (3 - 0) \hat{i} + (2 - 1) \hat{j} + (1 - 2) \hat{k} = 3\hat{i} + \hat{j} - \hat{k}$$

$$\vec{AC} = (1 - 0) \hat{i} + (2 - 1) \hat{j} + (1 - 2) \hat{k} = \hat{i} + \hat{j} - \hat{k}$$

$$\vec{AD} = (5 - 0) \hat{i} + (5 - 1) \hat{j} + (6 - 2) \hat{k} = 5\hat{i} + 4\hat{j} + 4\hat{k}$$

$$\text{Volume of tetrahedron} = \frac{1}{6} [\vec{AB} \cdot \vec{AC} \times \vec{AD}]$$

$$\begin{aligned} &= \frac{1}{6} \begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & -1 \\ 5 & 4 & 4 \end{vmatrix} = \frac{1}{6} \{3(4 + 4) - 1(4 + 5) - 1(4 - 5)\} \\ &= \frac{1}{6} (24 - 9 + 1) = \frac{16}{6} = \frac{8}{3} \end{aligned}$$