10th Class 2016

Math (Science)	Group-I	PAPER-II
Time: 20 Minutes	(Objective Type)	Max. Marks: 15

Note: Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1- If $\frac{a}{b}=\frac{c}{d}$, then componendo property is:

(a) $\frac{a}{a+b}=\frac{c}{c+d}$ √ (b) $\frac{a}{a-b}=\frac{c}{c-d}$

(c) $\frac{ad}{bc}$ (d) $\frac{a-b}{b}=\frac{c-d}{d}$

2- Point $(-1, 4)$ lies in the quadrant:

(a) I (b) II √

(c) III (d) IV

3- The solution set of equation $4x^2-16=0$ is:

(a) $\{\pm 4\}$ (b) $\{4\}$

(c) $\{\pm 2\}$ √ (d) $\{2\}$

4- The mode in the data 1, 3, 5, 3, 7, 9 is:

(a) 1 (b) 3 √

(c) 5 (d) 7

5- $\sec^2\theta =$ - - - - :

(a) $1-\sin^2\theta$ (b) $1+\tan^2\theta$ √

(c) $1+\cos^2\theta$ (d) $1-\tan^2\theta$

6- The semi-circumference and the diameter of a circle both subtend a central angle of:

(a) 90° (b) 180° √

(c) 270° (d) 360°

7- A complete circle is divided into:

(a) 90° (b) 180°

(c) 270° (d) 360° √

8- $\sin^2\theta + \cos^2\theta =$ ----- :

(a) $\sin\theta$ (b) $\cos\theta$
(c) 1 ✓ (d) 2

9- If $\frac{u}{v} = \frac{v}{w} = k$, then:

(a) $u = wk^2$ ✓ (b) $u = vk^2$
(c) $u = w^2k$ (d) $u = v^2k$

10- Two tangents drawn to a circle from a point outside it are of ----- in length:

(a) Half (b) Equal ✓
(c) Double (d) Triple

11- Angle inscribed in a semi-circle is:

(a) $\frac{\pi}{2}$ ✓ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{5}$

12- If A and B are disjoint sets, then $A \cup B$ is equal to:

(a) A (b) B
(c) ϕ (d) $B \cup A$ ✓

13- Product of cube roots of unity is:

(a) 0 (b) 1 ✓
(c) −1 (d) 3

14- $\frac{x^3 + 1}{(x-1)(x+2)}$ is ----- :

(a) A proper fraction (b) An improper fraction ✓
(c) An identity (d) A constant term

15- The discriminate of $ax^2 + bx + c = 0$ is:

(a) $b^2 - 4ac$ ✓ (b) $b^2 + 4ac$
(c) $-b^2 + 4ac$ (d) $-b^2 - 4ac$

10th Class 2016

Math (Science)	Group-I	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: 12

(i) Define radical equation.

Ans An equation involving expression under the radical sign is called a radical sign.

(ii) Solve by factorization: $5x^2 = 15$

Ans

$$5x^2 = 15$$

Dividing both sides by 5

$$\frac{5x^2}{5} = \frac{15}{5}$$

$$x^2 = 3$$

By taking under root both sides, we get

$$\sqrt{x^2} = \sqrt{3}$$

$$x = \pm\sqrt{3}$$

(iii) Find the discriminant of the following equation:

$$6x^2 - 8x + 3 = 0$$

Ans

$$6x^2 - 8x + 3 = 0$$

Discriminant $= b^2 - 4ac$

where,

$a = 6, b = -8, c = 3$, so

$$\begin{aligned}\text{Discriminant} &= (-8)^2 - 4(6)(3)\\ &= 64 - 72\\ &= -8\end{aligned}$$

(iv) Without solving, find the sum and the product of the roots of the equation:

$$7x^2 - 5mx + 9n = 0$$

Ans Here, $a = 7$, $b - 5m$, $c = 9n$

Sum of the roots $= \frac{-b}{a}$

$= \frac{-(-5m)}{7}$

$= \frac{5m}{7}$

Product of the roots $= \frac{c}{a}$

$= \frac{9n}{7}$

(v) Write quadratic equation having following roots: 2, – 6

Ans The given roots are: 2, –6

Thus,

Sum of the roots $= 2 + (-6)$

$= 2 - 6$

$S = -4$

Product of the roots $= 2(-6)$

$P = -12$

The quadratic equation having the known roots:

$$x^2 - Sx + P = 0$$

By putting 'S' and 'P', we get

$$x^2 - (-4)x + (-12) = 0$$

$$x^2 + 4x - 12 = 0$$

(vi) Find w^2, if: $\omega = \frac{-1 + \sqrt{-3}}{2}$

Ans If $\omega = \frac{-1 + \sqrt{-3}}{2}$

Then by the properties of cube roots of unity

$$\omega^2 = \frac{-1 - \sqrt{3}}{2}$$

(vii) Define direct variation.

Ans If two quantities are related in such a way that increase (decrease) in one quantity causes increase (decrease) in the other quantity then this variation is called direct variation.

(viii) Find a mean proportional between: 20, 45.

Ans

$$20 : x :: x : 45$$
$$x \cdot x = 20(45)$$
$$x^2 = 900$$
$$\sqrt{x^2} = \sqrt{900}$$
$$x = \pm 30$$

(ix) If a : b = c : d then prove that $\frac{4a + 5b}{4a - 5b} = \frac{4c + 5d}{4c - 5d}$.

Ans Since, $a : b = c : d$

$$\frac{a}{b} = \frac{c}{d}$$

Multiplying both side by $\frac{4}{5}$, we get

$$\frac{4a}{5b} = \frac{4c}{5d}$$

Then using componendo dividenothem.

$$\frac{4a + 5b}{4a - 5b} = \frac{4c + 5d}{4c - 5d}$$

Thus proved.

3. Write short answers to any SIX (6) questions: 12

(i) What are partial fractions?

Ans Partial fractions can be define as:

Decomposition of resultant fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$, when

(a) D(x) consists of non-repeated linear factors.
(b) D(x) consists of repeated linear factors.
(c) D(x) consists of non-repeated linear factors.
(d) D(x) consists of repeated irreducible quadratic factors.

(ii) Resolve into partial fractions: $\frac{x - 2}{(x + 2)(x + 3)}$.

$$\frac{x-2}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

Ans Multiplying both sides with $(x+2)(x+3)$,

$$\frac{(x-2)}{(x+2)(x+3)}(x+2)(x+3) = \left(\frac{A}{x+2} + \frac{B}{x+3}\right)(x+2)(x+3)$$

$$x-2 = \frac{A(x+3)+B(x+2)}{(x+2)(x+3)}(x+2)(x+3)$$

$$x-2 = A(x+3)+B(x+2) \qquad \text{(i)}$$

For finding 'A':

Let $x+2=0$

$x=-2$

By putting in (i),

$$-2-2 = A(-2+3)+B(-2+2)$$

$$-4 = A(1)+0$$

$\Rightarrow$ $A=-4$

For finding 'B':

Let $x+3=0$

$x=-3$

$$-3-2 = A(-3+3)+B(-3+2)$$

$$-5 = 0+B(-1)$$

$$-5 = -B$$

$\Rightarrow$ $B=5$

So,

$$\frac{x-2}{(x+2)(x+3)} = \frac{-4}{x+2} + \frac{5}{x+3}$$

(iii) Write all subsets of the set: {a, b}

Ans Let, $A=\{a,b\}$

All subsets of the above set:

$\{\}, \{a\}, \{b\}, \{a,b\}$

(iv) If $X=\phi$, $Y=Z^+$ then find $X\cap Y$.

Ans Given,

$$X\cap Y = \phi \cap \{0, 1, 2, 3, ---\}$$

$$= \phi$$

(v) **If $A = \{a, b\}$ and $B = \{c, d\}$ then find $B \times A$.**

Ans $B \times A = \{c, d\} \times \{a, b\}$
$= \{(c, a), (c, b), (d, a), (d, b)\}$

(vi) **Find the set X and Y if $X \times Y = \{(a, a), (b, a), (c, a), (d, a)\}$**

Ans Given,
$X \times Y = \{(a, a), (b, a), (c, a), (d, a)\}$
1^{st} element of each relation $= X = \{a, b, c, d\}$
2^{nd} element of each relation $= Y = \{a\}$

(vii) **Define class limit.**

Ans The minimum and the maximum values defined for a class or group are called class limits.

(viii) **Define arithmetic mean.**

Ans Arithmetic mean is a measure that determines a value of the variable under study by dividing the sum of all values of the variable by their number.

(ix) **Find the arithmetic mean by direct method:**
12, 14, 17, 20, 24, 29, 35, 45

Ans Let $x = 12, 14, 17, 20, 24, 29, 35, 45$

$$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{12 + 14 + 17 + 20 + 24 + 29 + 35 + 45}{8}$$

$$= \frac{196}{8}$$

$$\boxed{\bar{x} = 24.5}$$

4. Write short answers to any SIX (6) questions: 12

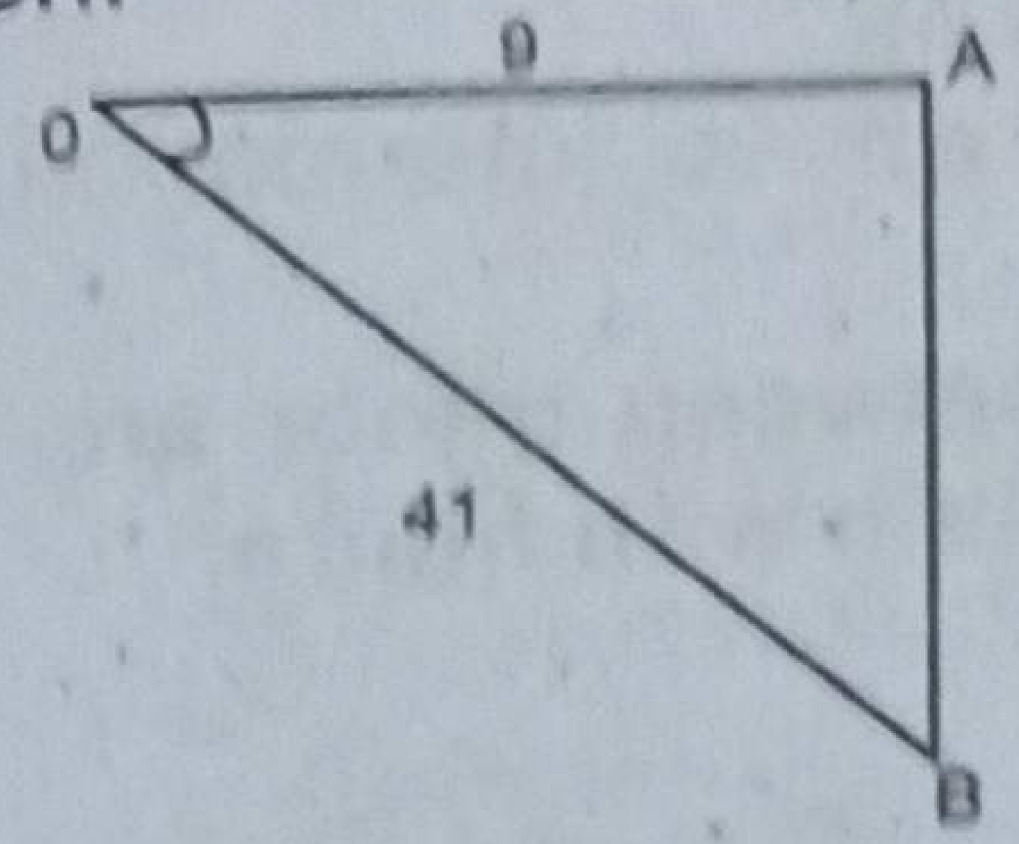
(i) **How many minutes are in two right angles?**

Ans As we know that:
1 degree = 60 minutes.
Two right angles have 180 degrees
Thus
Two right angles = 180×60 minutes

$= 10,800$ minutes

(ii) Find $\tan\theta$ when $\cos\theta = \frac{9}{41}$ and θ terminal side of the angle θ is in fourth quadrant.

Ans Given condition:



By Pythagoras theorem,

$$(m\overline{AB})^2 = (m\overline{OB})^2 - (m\overline{OA})^2$$

$$m\overline{AB} = \sqrt{(m\overline{OB})^2 - (m\overline{OA})^2}$$

$$= \sqrt{(41)^2 - (9)^2}$$

$$= \sqrt{1681 - 81} = \sqrt{1600}$$

$$m\overline{AB} = 40$$

So, $\tan\theta = \frac{40}{9}$

(iii) If $l = 4$ cm $\theta = \frac{1}{4}$ rad then find r.

Ans As we know that:

$$l = r\theta$$

$$\Rightarrow \quad r = \frac{l}{\theta}$$

$$r = \frac{4}{\frac{1}{4}}$$

$$= 4 \div \frac{1}{4}$$

$$= 4 \times 4$$

$$\boxed{r = 16}$$

(iv) Whether the triangle with sides 8 cm, 15 cm and 17 cm is acute, obtuse or right angled?

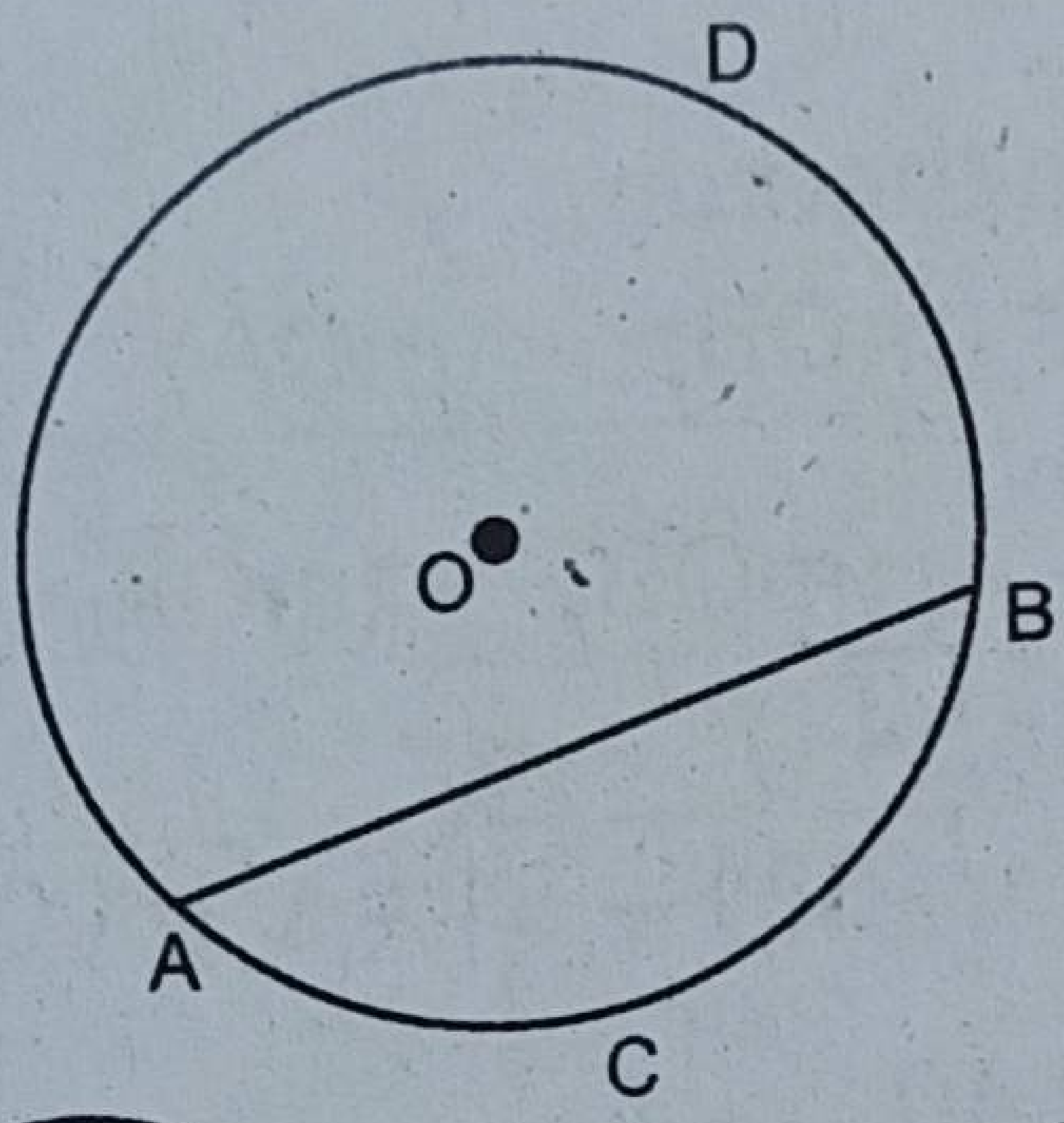
Ans By Pythagoras Theorem,

$$(17)^2 = (15)^2 + (8)^2$$
$$289 = 225 + 64$$
$$289 = 289$$

Prove that the above information is for right angled triangle.

(v) Differentiate between minor arc and major arc of a circle and explain with figure.

Ans



Smaller arc $\widehat{ACB}$ is called minor arc and greater arc $\widehat{ADB}$ is called major arc.

(vi) Define tangent of a circle.

Ans A tangent to a circle is the straight line which touches the circumference at one point only.

(vii) Define circumference of a circle.

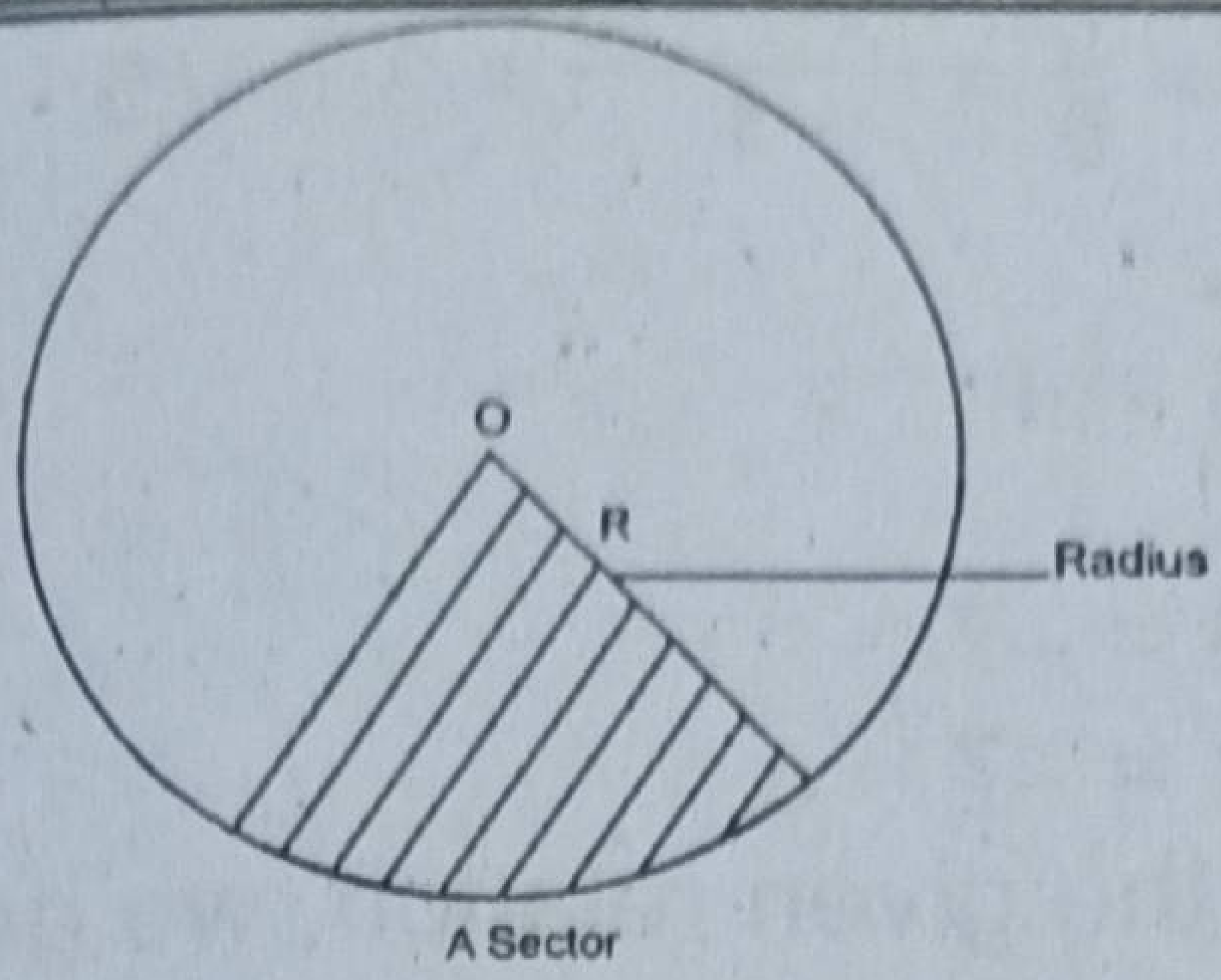
Ans The length of the boundary of the circle is called the circumference.

(viii) What is meant by cyclic quadrilateral?

Ans A quadrilateral is called cyclic when a circle can be drawn through its four vertices.

(ix) Define and draw the sector of a circle.

Ans A part of the circle bounded by the two radii and an arc is called sector of a circle.



(Part-II)

NOTE: Attempt any THREE (3) questions. But question 9 is compulsory.

Q.5.(a) Solve the equation: (4)

$$2x + 5 = \sqrt{7x + 16}$$

Ans $2x + 5 = \sqrt{7x + 16}$

By taking square both sides, we get

$$(2x + 5)^2 = (\sqrt{7x + 16})^2$$
$$(2x)^2 + (5)^2 + 2(2x)(5) = 7x + 16$$
$$4x^2 + 25 + 20x - 7x - 16 = 0$$
$$4x^2 + 13x + 9 = 0$$

By solving with factorization method,

$$4x^2 + 4x + 9x + 9 = 0$$
$$4x(x + 1) + 9(x + 1) = 0$$
$$4x + 9 = 0 \quad ; \quad x + 1 = 0$$
$$4x = -9 \quad ; \quad \boxed{x = -1}$$
$$\boxed{x = \frac{-9}{4}}$$

(b) Find m, if the equation $x^2 + 7x + 3m - 5 = 0$, satisfy the relation $3\alpha - 2\beta = 4$. (4)

Ans $x^2 + 7x + 3m - 5 = 0$

Let α, β, be the roots of equation

$$S = \alpha + \beta = \frac{-b}{a} = \frac{-7}{1} = -7 \qquad \text{(i)}$$

$$P = \alpha\beta = \frac{c}{a} = \frac{3m-5}{1} = 3m-5 \quad \text{(ii)}$$

But we have,

$$3\alpha - 2\beta = 4$$

From (i)

$$\alpha + \beta = -7$$
$$\beta = -7 - \alpha$$

By putting in the given relation, we get

$$3\alpha - 2(-7-\alpha) = 4$$
$$3\alpha + 14 + 2\alpha = 4$$
$$5\alpha = 4 - 14$$
$$\alpha = \frac{-10}{5}$$
$$\alpha = -2$$

By putting in (i), we get

$$\alpha + \beta = -7$$
$$-2 + \beta = -7$$
$$\beta = -7 + 2$$
$$\beta = -5$$

By putting the values of α and β in (ii), we get

$$(-2)(-5) = 3m - 5$$
$$10 = 3m - 5$$
$$\Rightarrow \quad 3m = 15$$
$$m = 5$$

Q.6.(a) If a : b : : c : d (a, b, c, d) ≠ 0 then prove that $\frac{a}{b} = \sqrt{\frac{a^2+c^2}{b^2+d^2}}$. (4)

Ans Given,

$$a : b :: c : d$$
$$\frac{a}{b} = \frac{c}{d}$$

Let, $\frac{a}{b} = \frac{c}{d} = k$

Then,

$$a = b\,k \quad \text{(i)}$$
$$c = d\,k \quad \text{(ii)}$$

Again, Given

$$\frac{a}{b} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}}$$

$$\text{L.H.S} = \frac{a}{b} \quad \text{(iii)}$$

By putting (i) in (iii), we get

$$= \frac{bk}{b}$$
$$= k$$

$$\text{R.H.S} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}} \quad \text{(iv)}$$

By putting (i) and (ii) in (iv), we get

$$= \sqrt{\frac{(bk)^2 + (dk)^2}{b^2 + d^2}}$$
$$= \sqrt{\frac{b^2k^2 + d^2k^2}{b^2 + d^2}}$$
$$= \sqrt{\frac{k^2(b^2 + d^2)}{(b^2 + d^2)}}$$
$$= \sqrt{k^2}$$
$$= k$$

So,

$$\text{L.H.S} = \text{R.H.S}$$
$$\frac{a}{b} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}} \qquad \text{Proved.}$$

(b) Resolve into partial fractions: $\frac{7x - 9}{(x + 1)(x - 3)}$. (4)

Ans

$$\frac{7x - 9}{(x + 1)(x - 3)} = \frac{A}{x + 1} + \frac{B}{x - 3} \quad \text{(i)}$$

$$\frac{7x - 9}{(x + 1)(x - 3)} = \frac{A(x - 3) + B(x + 1)}{(x + 1)(x - 3)}$$

Multiply both sides by $(x + 1)(x - 3)$

$$(x + 1)(x - 3)\frac{7x - 9}{(x + 1)(x - 3)} = \frac{A(x - 3) + B(x + 1)}{(x + 1)(x - 3)} \times (x + 1)(x - 3)$$

$$7x - 9 = A(x - 3) + B(x + 1) \quad \text{(ii)}$$

For finding 'A',

Let $x + 1 = 0$

$x = -1$

By putting $x = -1$ in (ii), we get

$$7(-1) - 9 = A(-1 - 3) + B(-1 + 1)$$

$$-7 - 9 = A(-4) + 0$$

$$\Rightarrow \quad -4A = -16$$

$$A = \frac{-16}{-4}$$

$$\boxed{A = 4}$$

For finding 'B',

Put $x - 3 = 0$

$x = 3$

By putting $x = 3$ in (ii), we get

$$7(3) - 9 = A(3 - 3) + B(3 + 1)$$

$$21 - 9 = 0 + 4B$$

$$12 = 4B$$

$$\Rightarrow \quad B = \frac{12}{4}$$

$$\boxed{B = 3}$$

Put the values of A and B in (i),

$$\frac{7x - 9}{(x + 1)(x - 3)} = \frac{4}{x + 1} + \frac{3}{x - 3}$$

Q.7.(a) If $y = \{-2, 1, 2\}$, then make two binary relations for $y \times y$? Also find their domain and range. (4)

Ans

$$y = \{-2, 1, 2\}$$

$$y \times y = \{-2, 1, 2\} \times \{-2, 1, 2\}$$

$$y \times y = \{(-2, -2), (-2, 1), (-2, 2), (1, -2), (1, 1), (1, 2), (2, -2), (2, 1), (2, 2)\}$$

$$R_1 = \{(-2, -2), (-2, 1), (1, 2), (2, 2)\}$$

Dom $R_1 = \{-2, 1, 2\}$

Range $R_1 = \{-2, 1, 2\}$

Similarly,

$$R_2 = \{(-2, 1), (1, 1), (-2, 2)\}$$

Dom $R_2 = \{-2, 1\}$

Range $R_2 = \{1, 2\}$

(b) The following frequency distribution shows weights of boys in kilogram. Compute Median: (4)

Class Interval	1 – 3	4 – 6	7 – 9	10 – 12	13 – 15	16 – 18	19 – 21
Frequency	2	3	5	4	6	2	1

Ans

Class Interval	Frequency	C – B	C – F
1 – 3	2	0.5 – 3.5	2
4 – 6	3	3.5 – 6.5	5
7 – 9	5	6.5 – 9.5	10
10 – 12	4	9.5 – 12.5	14
13 – 15	6	12.5 – 15.5	20
16 – 18	2	15.5 – 18.5	22
19 – 21	1	18.5 – 21.5	23
	23		

For obtaining median class,

$$\frac{\Sigma f}{2} = \frac{23}{2} = 11.5$$

As $\frac{\Sigma f}{2} = 11.5$, so with respect to C – F, 4^{th} row is the median class.

Thus,

$$\text{Median} = \tilde{X} = l + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

Here,

l = lower class boundary of the median class = 9.5

h = class interval of the median class = 3

f = frequency of the median class = 4

C = Cumulative frequency preceding the median class = 10

So, $\text{Median} = 9.5 + \frac{3}{4}(11.5 - 10)$

$= 9.5 + 1.125$

$\boxed{\text{Median} = 10.63}$

Q.8.(a) Verify that: (4)

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{\sin\theta}{1-\cos\theta}$$

Ans L.H.S $= \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$

$$= \sqrt{\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}}$$

$$= \sqrt{\frac{(1)^2-(\cos\theta)^2}{(1-\cos\theta)^2}}$$

$$= \frac{\sqrt{1-\cos^2\theta}}{1-\cos\theta}$$

As we know that

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta = 1 - \cos^2\theta$$

So that,

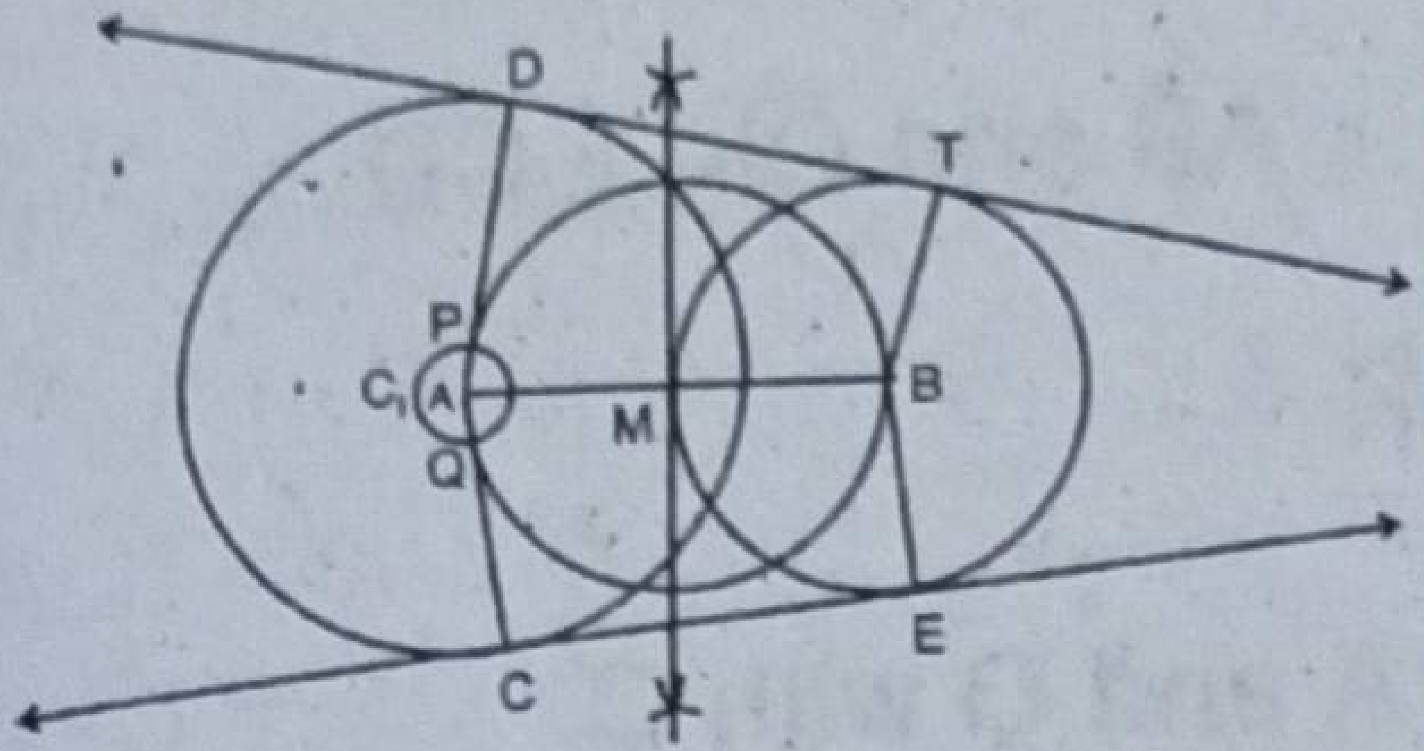
$$= \frac{\sqrt{\sin^2\theta}}{1-\cos\theta}$$

$$= \frac{\sin\theta}{1-\cos\theta}$$

= R.H.S Proved.

(b) **Draw two common tangents to two intersecting circle of radii 3 cm and 4 cm.**

Ans

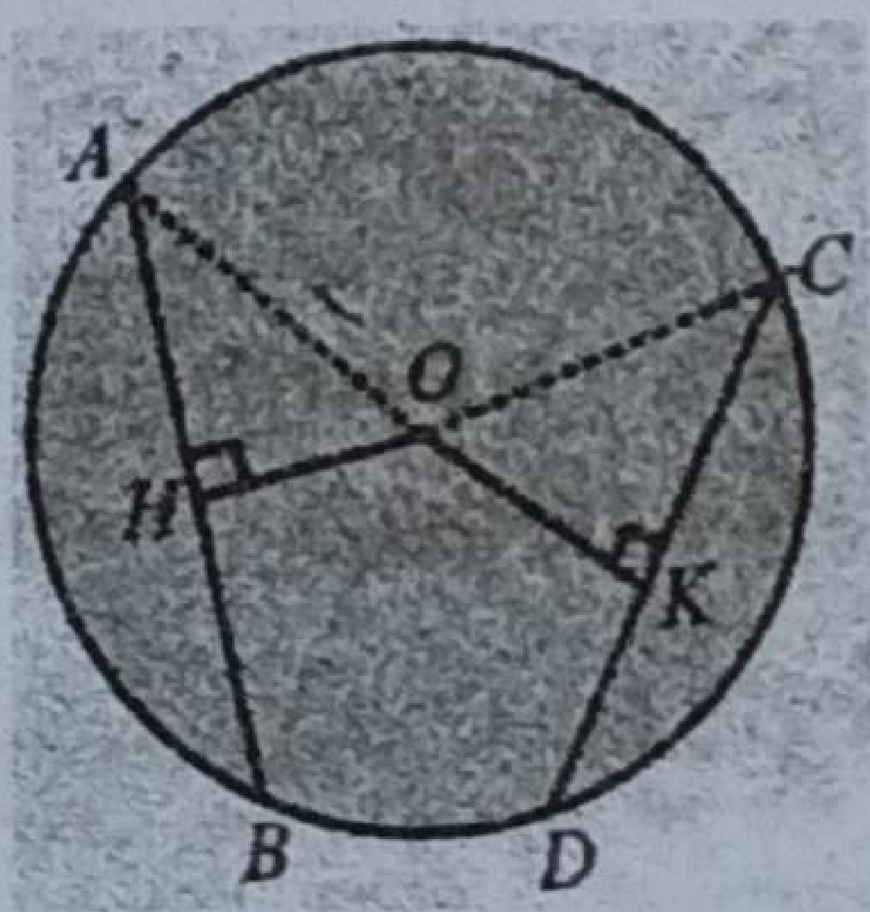


Procedure:

1. Take a line $\overline{AB} = 3 + 4 = 7$ cm.
2. Take A as centre and draw circle of 4 cm radius.
3. Take B as centre and draw circle of 3 cm radius which cut the first circle at two points.
4. Take M as the middle point of $\overline{AB}$ and draw the circle of $\overline{MA}$ radius.
5. Now again consider A as centre and draw the circle of $(4 - 3 =)$ 1 cm radius which cut the other circle at P and Q.
6. Join A with P and Q and join the big circle with C and C_1.
7. Take $\overline{BT} \parallel \overline{AD}$ and $\overline{BE} \parallel \overline{AC}$.
8. Join D with T and C with E.
9. $\overleftrightarrow{DT}$ and $\overleftrightarrow{CE}$ are the required common tangents.

Q.9. Prove that if two chords of a circle are congruent then they will be equidistant from the centre. (4)

Ans



Given:

$\overline{AB}$ and $\overline{CD}$ are two equal chords of a circle with centre at O.
So that $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$.

To prove:

$m\overline{OH} = m\overline{OK}$

Construction:

Join O with A and O with C.
So that we have $\angle rt\Delta^s$ OAH and OCK.

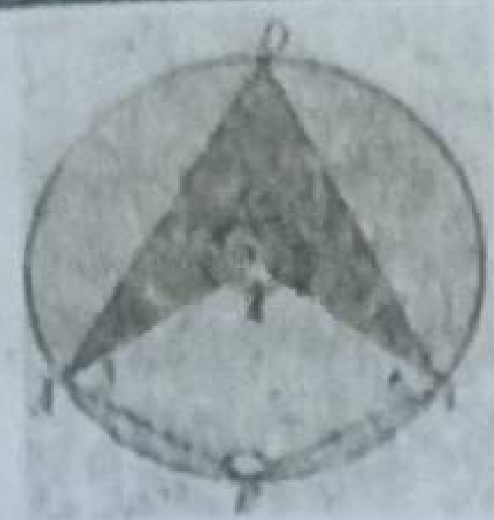
Proof:

Statements	Reasons
$\overline{OH}$ bisects chord $\overline{AB}$	$\overline{OH} \perp \overline{AB}$ By Theorem 3
i.e., $m\overline{AH} = \frac{1}{2} m\overline{AB}$ (i)	
Similarly $\overline{OK}$ bisects chord $\overline{CD}$	$\overline{OK} \perp \overline{CD}$ By Theorem 3
i.e., $m\overline{CK} = \frac{1}{2} m\overline{CD}$ (ii)	
But $m\overline{AB} = m\overline{CD}$ (iii)	Given
Hence $m\overline{AH} = m\overline{CK}$ (iv)	Using (i), (ii) & (iii)
Now in $\angle rt\ \Delta^s$ OAH $\leftrightarrow$ OCK	Given $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$
hyp $\overline{OA}$ = hyp $\overline{OC}$	Radii of the same circle
$m\overline{AH} = m\overline{CK}$	Already proved in (iv)
$\therefore$ $\Delta OAH \cong \Delta OCK$	H.S postulate
$\Rightarrow$ $m\overline{OH} = m\overline{OK}$	

OR

Prove that the opposite angles of any quadrilateral inscribed in a circle are supplementary.

Ans



Given:
ABCD is a quadrilateral inscribed in a circle with centre O.

To prove:
$$\begin{cases} m\angle A + m\angle C = 2\angle rts \\ m\angle B + m\angle D = 2\angle rts \end{cases}$$

Construction:
Draw $\overline{OA}$ and $\overline{OC}$.
Write $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$ as shown in the figure.

Statements	Reasons
Standing on the same arc ADC, $\angle 2$ is a central angle. Whereas $\angle B$ is the circumangle	Arc ADC of the circle with centre O.
$\therefore\quad m\angle B = \frac{1}{2}(m\angle 2)$ (i)	By theorem 1
Standing on the same arc ABC, $\angle 4$ is a central angle whereas $\angle D$ is the circumangle	Arc ABC of the circle with centre O.
$\therefore\quad m\angle D = \frac{1}{2}(m\angle 4)$ (ii)	By theorem 1
$\Rightarrow\quad m\angle B + m\angle D = \frac{1}{2} m\angle 2 + \frac{1}{2} m\angle 4$ $= \frac{1}{2}(m\angle 2 + m\angle 4) = \frac{1}{2}$ (Total central angle) *i.e.*, $m\angle B + m\angle D = \frac{1}{2}(4\angle rt)$ $= 2\angle rt$ Similarly $m\angle A + m\angle C = 2\angle rt$	Adding (i) and (ii)