

Inter (Part-II) 2015

Mathematics**Group-II****PAPER: II****Time: 30 Minutes****(OBJECTIVE TYPE)****Marks: 20**

Note: Four possible answers, A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1- If $\lim_{x \rightarrow 0} \frac{f(x) - 1}{x} = \ln a ; a > 0$, then:

- (a) a^{-x} (b) a^x
 (c) $e^{-x} \checkmark$ (d) e^x

2- $\lim_{x \rightarrow \infty} \left(1 + \frac{x}{2}\right)^{1/x}$ equals:

- (a) $e \checkmark$ (b) e^{-1}
 (c) e^2 (d) \sqrt{e}

3- $\frac{d}{dx} [\ln x]$ is equal to:

- (a) x (b) $\frac{1}{x} \checkmark$
 (c) x^2 (d) $\frac{1}{x^2}$

4- Derivative of $\sin h^{-1} x$ w.r.t x equals:

- (a) $\frac{1}{\sqrt{1-x^2}}$ (b) $\frac{-1}{\sqrt{1-x^2}}$
 (c) $\frac{1}{\sqrt{1+x^2}} \checkmark$ (d) $\frac{-1}{\sqrt{1+x^2}}$

5- $\frac{d}{dx} (e^{\cos x})$ equals:

- (a) $-\sin x e^{\cos x} \checkmark$ (b) $\sin x e^{\cos x}$
 (c) $\cos x e^{\sin x}$ (d) $-\cos x e^{\sin x}$

6- If $f(x + h) = \cos (x + h)$, then $f'(x)$ equals:

- (a) $\cos x$ (b) $-\cos x$
 (c) $\sin x$ (d) $-\sin x \checkmark$

7- If $y = \sec\left(\frac{3\pi}{2} - x\right)$, then y_1 equals:

- (a) cosec $x \cot x$ ✓ (b) -cosec $x \cot x$
 (c) sec $x \tan x$ (d) -sec $x \tan x$

8- $\int \sin x \, dx$ is equal to:

- (a) cos x (b) -cos x ✓
 (c) sin x (d) -sin x

9- $\int \frac{1}{1+x^2} \, dx$ is equal to:

- (a) $\tan^{-1} x$ ✓ (b) $\tan^{-1} x^2$
 (c) $\cot^{-1} x$ (d) $\cot^{-1} x^2$

10- $\int_a^b x \, dx$ equals:

- (a) $\frac{b-a}{2}$ (b) $\frac{b+a}{2}$
 (c) $\frac{b^2 - a^2}{2}$ ✓ (d) $\frac{b^2 + a^2}{2}$

11- $\int \frac{1}{x \cdot \ln x} \, dx$ equals:

- (a) $\ln(\ln x)$ ✓ (b) $\ln x$
 (c) $x \ln x$ (d) $\frac{\ln x}{x}$

12- $\int_1^4 3\sqrt{x} \, dx$ is equal to:

- (a) 1 (b) 4
 (c) 14 ✓ (d) 41

13- $\int e^{2x} (-\sin x + 2 \cos x) \, dx$ equals:

- (a) $e^{2x} \sin x$ (b) $e^{2x} \cos x$ ✓
 (c) $-e^{2x} \sin x$ (d) $-e^{2x} \cos x$

14- Slope intercept form of line equals:

- (a) $y - y_1 = m(x - x_1)$ (b) $\frac{x}{a} + \frac{y}{b} = 1$
 (c) $x \cos \theta + y \sin \theta = p$ (d) $y = mx + c$ ✓

- 15- Point of intersection of lines $x - 2y + 1 = 0$ and $2x - y + 2 = 0$ equals:
- (a) $(1, 0)$
 - (b) $(0, 1)$
 - (c) $(-1, 0)$ ✓
 - (d) $(0, -1)$
- 16- A function which is to be maximized or minimized is called:
- (a) Exponential function
 - (b) Linear function
 - (c) Quadratic function
 - (d) Objective function ✓
- 17- Equation of axis of parabola $x^2 = 4ay$ is:
- (a) $x = 0$ ✓
 - (b) $x = a$
 - (c) $y = 0$
 - (d) $y = a$
- 18- Length of tangent from $(0, 1)$ to $x^2 + y^2 + 6x - 3y + 3 = 0$ is:
- (a) 2
 - (b) 3
 - (c) 4
 - (d) 1 ✓
- 19- Moment of force \underline{F} about (\underline{r}) is:
- (a) $\underline{r} \times \underline{F}$ ✓
 - (b) $\underline{F} \times \underline{r}$
 - (c) $\underline{r} \cdot \underline{F}$
 - (d) $\underline{F} \cdot \underline{r}$
- 20- $2\underline{i} \cdot (2\underline{j} \times \underline{k})$ equals:
- (a) 4 ✓
 - (b) 3
 - (c) 2
 - (d) 1

Inter (Part-II) 2015

Mathematics

Group-II

PAPER: II

Time: 2.30 Hours

(SUBJECTIVE TYPE)

Marks: 80

SECTION-I

2. Write short answers to any EIGHT (8) questions: 16

(i) If $f(x) = \sin x$ then find $\frac{f(a+h) - f(a)}{h}$.

$$\text{Ans} \quad \frac{f(a+h) - f(a)}{h} = \frac{\sin(a+h) - \sin a}{h}$$

$$= \frac{2 \cos(a + \frac{h}{2}) \cdot \sin \frac{h}{2}}{h}$$

(ii) Evaluate $\lim_{x \rightarrow 0} \frac{\sin x^0}{x}$.

$$\text{Ans} \quad \lim_{x \rightarrow 0} \frac{\sin x^0}{x} = \lim_{x \rightarrow 0} \sin \frac{\frac{\pi x}{180}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\sin \frac{\pi x}{180}}{x} \cdot \frac{\pi}{180} = (1) \frac{\pi}{180} = \frac{\pi}{180}$$

(iii) Define continuity.

Ans A function f is said to be continuous at a number c iff (i) $f(c)$ is defined (ii) $\lim_{x \rightarrow c} f(x)$ exists (iii) $\lim_{x \rightarrow c} f(x) = f(c)$.

(iv) Differentiate by definition x^2 .

Ans $y = x^2 \Rightarrow y + \delta y = (x + \delta x)^2, \delta y = 2x \delta x + \delta x^2$

$$\frac{\delta y}{\delta x} = 2x + \delta x, \quad \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} 2x + \delta x \Rightarrow \frac{dy}{dx} = 2x$$

(v) Differentiate w.r.t. x $\frac{x^2 + 1}{x^2 - 3}$.

Ans $y = \frac{x^2 + 1}{x^2 - 3}$

$$\frac{dy}{dx} = \frac{(x^2 - 3) \frac{d}{dx}(x^2 + 1) - (x^2 + 1) \frac{d}{dx}(x^2 - 3)}{(x^2 - 3)^2}$$

$$= \frac{-8x}{(x^2 - 3)^2}$$

(vi) Differentiate $\sin x$ w.r.t $\cot x$.

Ans Let $y = \sin x$ and $z = \cot x$

$$\frac{dy}{dx} = \cos x \quad \text{and} \quad \frac{dz}{dx} = -\operatorname{cosec}^2 x$$

$$\frac{dy}{dz} = \frac{dy}{dx} \cdot \frac{dx}{dz}$$

$$\frac{dy}{dz} = \cos x - \frac{1}{\operatorname{cosec}^2 x} = -\cos x \sin^2 x$$

(vii) If $f(x) = \ln(e^x + e^{-x})$ then find $f'(x)$.

$$\text{Ans} \quad f(x) = \ln(e^x + e^{-x})$$

$$\Rightarrow f'(x) = \frac{1}{e^x + e^{-x}} \times \frac{d}{dx}(e^x + e^{-x})$$

$$f'(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\left(e^x - \frac{1}{e^x}\right)}{\left(e^x + \frac{1}{e^x}\right)} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

(viii) Find y_2 if $x^2 + y^2 = a^2$.

$$\text{Ans} \quad x^2 + y^2 = a^2$$

$$\Rightarrow 2x + 2y \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = y_1 = \frac{-x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\frac{dy}{dx}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$$

$$y_2 = \frac{d^2y}{dx^2} = \frac{-a^2}{y^3}$$

Expand Maclaurin's series and prove $e^x = 1 + x + \frac{x^2}{2!} + \dots$

(ix)

Expand Maclaurin's series and prove $e^x = 1 + x + \frac{x^2}{2!} + \dots$

$$+ \frac{x^3}{3!} + \dots$$

Ans →

Maclaurin series of $f(x)$ is:

$$f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + \dots$$

$$\text{If } f(x) = e^x, f'(0) = 1, f''(0) = 1, f'''(0) = 1, \dots$$

$$\therefore f(x) = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

Define increasing and decreasing function.

(x)

Ans → Define increasing and decreasing function.

then:

1. f is increasing on the interval (a, b) if $f(x_2) > f(x_1)$ whenever $x_2 > x_1$.

2. f is decreasing on the interval (a, b) if $f(x_2) < f(x_1)$ whenever $x_2 > x_1$.

(xi)

Find $\frac{dy}{dx}$ if $y = \ln \tan hx$.

Ans →

$$y = \ln \tan hx$$

$$\frac{dy}{dx} = \frac{1}{\tan hx} \cdot \frac{d}{dx}(\tan hx)$$

$$\frac{dy}{dx} = \frac{1}{\tan hx} \times \sec^2 hx = 2 \operatorname{cosec} h 2x$$

(xii)

Find $\frac{dy}{dx}$ if $xy + y^2 = 2$.

Ans →

$$xy + y^2 = 2$$

$$\Rightarrow \left[1 \cdot y + x \frac{dy}{dx} \right] + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-y}{x + 2y}$$

3. Write short answers to any EIGHT (8) questions: 16

(i) Use differential to approximate the value of $\cos 29^\circ$.

Ans →

Let $y = \cos x \Rightarrow \frac{dy}{dx} = -\sin x \Rightarrow dy = -\sin x dx$

Take $x = 30^\circ$ then $dx = -1 = -0.01746$ rad.

$$dy = -\sin 30^\circ \times (-0.1746) = 0.00873 \approx \delta y$$

$$\text{Now } y = \cos x \Rightarrow y + \delta y = \cos(x + \delta x)$$

$$\delta y = \cos(x + \delta x) - \cos x$$

$$\Rightarrow \cos(x + \delta x) = \delta x + \cos x$$

$$\Rightarrow \cos 29^\circ = \delta y + \cos 30^\circ = 0.008 > 3 + 0.866 \approx 0.874$$

(ii) Evaluate $\int x \sqrt{x^2 - 1} dx$.

Ans $\int x \sqrt{x^2 - 1} dx$

$$= \frac{1}{2} \int 2x (x^2 - 1)^{1/2} dx$$

$$= \frac{1}{3} (x^2 - 1)^{3/2} + c$$

(iii) Evaluate $\int \cos 3x \sin 2x dx$.

Ans $\int \cos 3x \sin 2x dx \Rightarrow \frac{1}{2} \int 2 \cos 3x \sin 2x dx$

$$= \frac{1}{2} \int [\sin 5x - \sin x] dx$$

$$= \frac{1}{2} \left[-\frac{\cos 5x}{5} + \cos x \right] + c$$

(iv) Evaluate $\int x \ln x dx$.

Ans $\int x \ln x dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x} dx = \frac{x^2}{2} \ln x - \frac{1}{2} \int x dx$

$$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + c$$

(v) Evaluate $\int \frac{2x}{1 - \cos x} dx$.

Ans $\int \frac{2x}{1 - \cos x} dx = \int \frac{2x dx}{2 \sin^2 \frac{x}{2}} = \int x \operatorname{cosec}^2 \frac{x}{2} dx$

$$= -x \cdot 2 \cot \frac{x}{2} + 2 \int \cot \frac{x}{2} dx$$

$$= -2x \cot \frac{x}{2} + 4 \ln \sin \frac{x}{2} + c$$

(vi) Evaluate $\int \frac{3-x}{1-x-6x^2} dx$.

Ans Consider,

$$\begin{aligned}\frac{3-x}{1-x-6x^2} &= \frac{3-x}{1-3x+2x-6x^2} \\&= \frac{3-x}{1(1-3x)+2x(1-3x)} \\&= \frac{3-x}{(1+2x)(1-3x)}\end{aligned}$$

We can write,

$$\frac{3-x}{(1+2x)(1-3x)} = \frac{A}{1+2x} + \frac{B}{1-3x}$$

Using the method of partial fractions, we get $A = \frac{7}{5}$, $B = \frac{8}{5}$.

Thus,

$$\begin{aligned}\frac{3-x}{(1+2x)(1-3x)} dx &= \frac{\frac{7}{5}}{1+2x} + \frac{\frac{8}{5}}{1-3x} \\&= \frac{7}{5(1+2x)} + \frac{8}{5(1-3x)}\end{aligned}$$

Thus,

$$\begin{aligned}\int \frac{(3-x) dx}{(1+2x)(1-3x)} &= \int \frac{7}{5(1+2x)} dx + \int \frac{8}{5(1-3x)} dx \\&= \frac{7}{5} \cdot \left(\frac{1}{2}\right) \int (1+2x)^{-1}(2) dx + \frac{8}{5} \left(\frac{-1}{3}\right) \int (1-3x)^{-1}(-3) dx \\&= \frac{7}{10} \ln |1+2x| - \frac{8}{15} \ln |1-3x| + C\end{aligned}$$

(vii) Find $\int_{-1}^3 (x^3 + 3x^2) dx$.

Ans $\int_{-1}^3 (x^3 + 3x^2) dx = \left[\frac{x^4}{4} + x^3 \right]_{-1}^3 = \left[\frac{x^4}{4} \right]_{-1}^3 + [x^3]_{-1}^3$
 $= \left[\frac{81}{4} - \frac{1}{4} \right] + [27 - (-1)] = 20 + 28$
 $= 48$

(viii) Evaluate $\int_0^{\pi/6} x \cos x dx$.

Ans $\int_0^{\pi/6} x \cos x dx = x \sin x \Big|_0^{\pi/6} - \int_0^{\pi/6} \sin x dx$

$$\begin{aligned}
 &= x \sin x + \cos x \Big|_0^{\pi/6} \\
 &= \left[\frac{\pi}{6} \sin \frac{\pi}{6} + \cos \frac{\pi}{6} \right] - (0 \sin 0 + \cos 0) \\
 &= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1
 \end{aligned}$$

- (ix) Find the area bounded by the curve $y = x^3 + 1$, the x-axis and the line $x = 2$.

Ans $x^3 + 1 = 0 \Rightarrow (x+1)(x^2 - x + 1) = 0$
 $x = -1$ is only real value

$$\begin{aligned}
 \text{Required area: } &\int_{-1}^2 (x^3 + 1) dx \\
 &= \frac{x^4}{4} + x \Big|_{-1}^2 = \frac{27}{4} \text{ sq. units}
 \end{aligned}$$

- (x) Solve the differential equation $\frac{dy}{dx} = \frac{y^2 + 1}{e^{-x}}$.

Ans $\frac{dy}{dx} = \frac{y^2 + 1}{e^{-x}}$

$$\Rightarrow \frac{dy}{y^2 + 1} = e^x dx$$

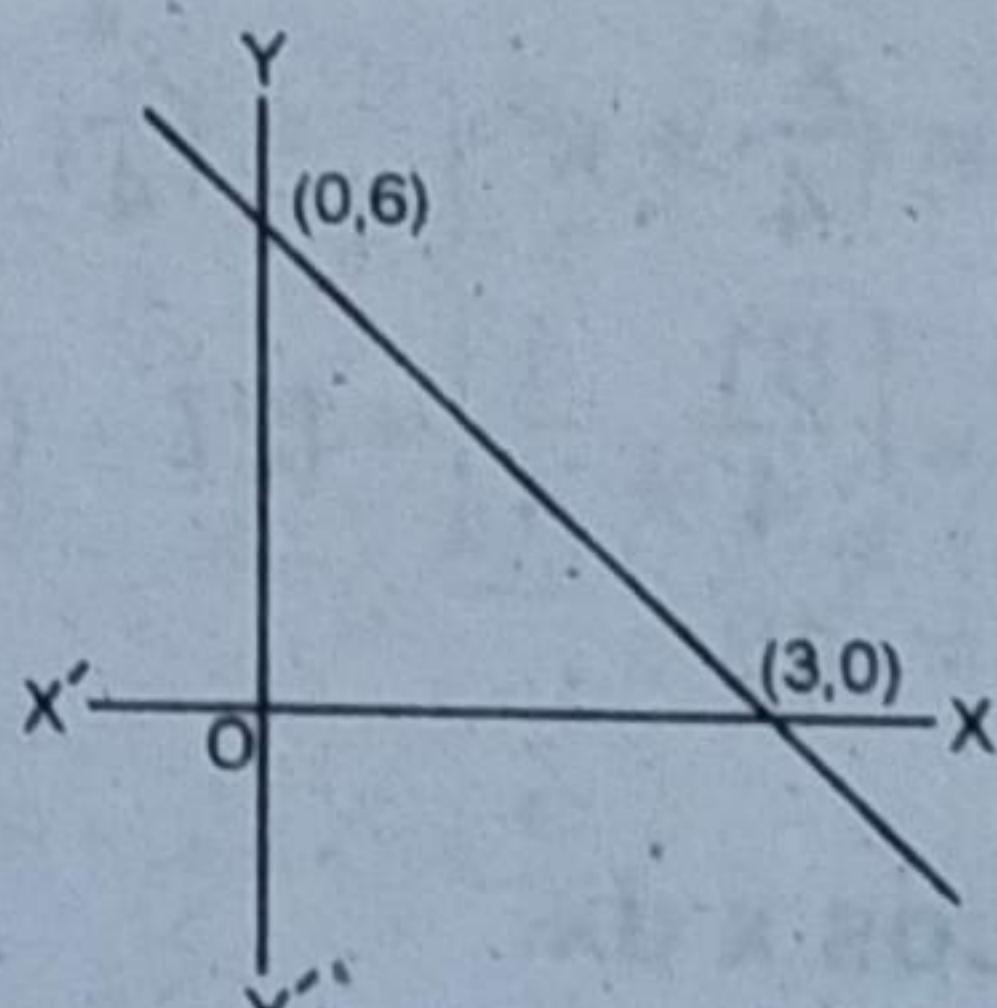
$$\Rightarrow \tan^{-1} y = e^x + c$$

$$\Rightarrow y = \tan(e^x + c)$$

- (xi) Graph the solution set of the inequality $2x + y \leq 6$.

Ans $2x + y \leq 6$

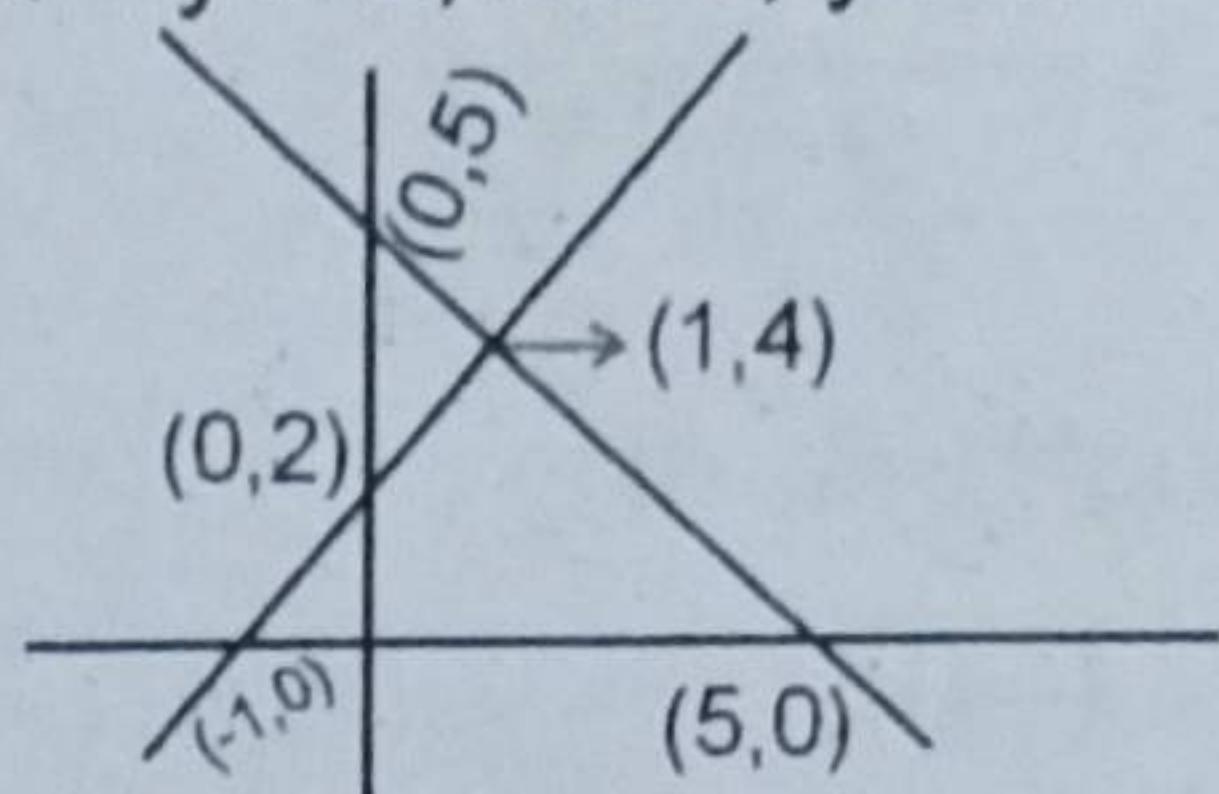
Two points on the associated equation $2x + y = 6$ are $(0, 6)$ and $(3, 0)$.



$\therefore 0 + 0 < 6$ solution lies towards the origin side.

- (xii) Find the corner points of inequalities: $x + y \leq 5$; $-2x + y \geq 2$; $x \geq 0$; $y \geq 0$.

Ans $x + y \leq 5; -2x + y \geq 2; x \geq 0; y \geq 0$



Two points on $x + y = 5$ are $(0, 5)$, $(5, 0)$; two points on $-2x + y = 2$ are $(-1, 0)$, $(0, 2)$ and drawing lines.

Finding corner points $(1, 4)$, $(0, 2)$, $(0, 5)$.

4. Write short answers to any NINE (9) questions: 18

- (i) Find point which divide A $(-6, 3)$ and B $(5, -2)$ internally in $2 : 3$.

Ans $k_1 = 2$ $k_2 = 3$

$$x = \frac{2 \times 5 + 3(-6)}{2 + 3}, \quad y = \frac{2(-2) + 3 \times 3}{2 + 3}$$

$$= \left(\frac{-8}{5}, 1 \right)$$

- (ii) Transform $5x - 12y + 39 = 0$ into two intercept form.

Ans $\frac{5x}{-39} - \frac{12y}{-39} = \frac{-39}{-39}$

$$\frac{x}{5} + \frac{y}{4} = 1$$

- (iii) Check whether $(-2, 4)$ lies above or below $4x + 5y - 3 = 0$.

Ans $(x, y) = (-2, 4)$

$$4x + 5y - 3 = 0$$

$$4(-2) + 5 \times 4 - 3$$

$$-8 + 20 - 3$$

$$9 > 0$$

$(-2, 4)$ lies above the line.

- (iv) Find equation of line through $(-4, -6)$ and perpendicular to the line having slope $\frac{-3}{2}$.

Ans Slope of given line $= \frac{-3}{2}$

$$\text{Slope of } \perp \text{ line} = \frac{-1}{\frac{-3}{2}} = \frac{2}{3}$$

Eq. of line through $(-4, -6)$

$$y - (-6) = \frac{2}{3}(x - (-4))$$

$$3y + 18 = 2x + 8$$

$$2x - 3y - 10 = 0$$

- (v) Find condition that the lines $y = m_1x + c_1$, $y = m_2x + c_2$, $y = m_3x + c_3$ are concurrent.

Ans $y - m_1x - c_1 = 0$ (i)

$$y - m_2x - c_2 = 0$$
 (ii)

$$y - m_3x - c_3 = 0$$
 (iii)

Lines (i), (ii) and (iii) will be concurrent, if

$$\begin{vmatrix} -m_1 & 1 & -c_1 \\ -m_2 & 1 & -c_2 \\ -m_3 & 1 & -c_3 \end{vmatrix} = 0$$

Apply $R_2 - R_1$ and $R_3 - R_1$

$$\begin{vmatrix} -m_1 & 1 & -c_1 \\ -m_2 + m_1 & 1 - 1 & -c_2 + c_1 \\ -m_3 + m_1 & 1 - 1 & -c_3 + c_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} -m_1 & 1 & -c_1 \\ -m_2 + m_1 & 0 & -c_2 + c_1 \\ -m_3 + m_1 & 0 & -c_3 + c_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} -m_2 + m_1 & -c_2 + c_1 \\ -m_3 + m_1 & -c_3 + c_1 \end{vmatrix} = 0$$

$$(-m_2 + m_1)(-c_3 + c_1) - (-m_3 + m_1)(-c_2 + c_1) = 0$$

$$(m_1 - m_2)(c_1 - c_3) = (m_1 - m_3)(c_1 - c_2)$$

is the required condition.

- (vi) Find centre and radius of circle $5x^2 + 5y^2 + 14x + 12y - 10 = 0$.

Ans $\frac{5x^2}{5} + \frac{5y^2}{5} + \frac{14x}{5} + \frac{12y}{5} - \frac{10}{5} = \frac{0}{5}$

$$x^2 + y^2 + \frac{14}{5}x + \frac{12}{5}y - 2 = 0$$

$$2g = \frac{14}{5}, \quad 2f = \frac{12}{5}, \quad c = -2$$

$$\text{centre } (-g, -f) \Rightarrow \left(\frac{-7}{5}, \frac{-6}{5} \right)$$

$$\begin{aligned}\text{Radius} &= \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{6}{5}\right)^2 - (-2)} \\ &= \frac{\sqrt{135}}{5}\end{aligned}$$

- (vii) Find length of tangent segment from $(-5, 4)$ to $5x^2 + 5y^2 - 10x + 15y - 131 = 0$.

Ans $\frac{5x^2}{5} + \frac{5y^2}{5} - \frac{10x}{5} + \frac{15y}{5} - \frac{131}{5} = \frac{0}{5}$

$$x^2 + y^2 - 2x + 3y - \frac{131}{5} = 0$$

$$(x, y) = (-5, 4)$$

Tangent length

$$\begin{aligned}&= \sqrt{(-5)^2 + 4^2 - 2(-5) + 3 \times 4 - \frac{131}{5}} \\ &= \sqrt{\frac{184}{5}}\end{aligned}$$

- (viii) Find equation of parabola with focus $(1, 2)$, vertex $(3, 2)$.

Ans Distance between focus and vertex a is:

$$\sqrt{(3 - 1)^2 + (2 - 2)^2} = 2$$

As the vertex is $(3, 2)$ so $h = 3$, $k = 2$, and $a = 2$

$$(y - 2)^2 = -4(2)(x - 3)$$

$$\text{So eq. } (y - k)^2 = -4a(x - h)$$

$$y^2 - 4y + 8x - 20 = 0$$

- (ix) Find foci, eccentricity of hyperbola $\frac{y^2}{4} - x^2 = 1$.

Ans $\frac{y^2}{4} - \frac{x^2}{1} = 1$

Centre $(0, 0)$

$$a^2 = 4 \Rightarrow a = 2$$

$$b^2 = 1 \Rightarrow b = 1$$

$$c^2 = a^2 + b^2$$

$$c^2 = 4 + 1$$

$$e = \frac{c}{a} = \frac{\sqrt{5}}{2}$$

$$c^2 = 5 \Rightarrow c = \sqrt{5}$$

$$\text{foci} = (0, \pm c) = (0, \pm \sqrt{5})$$

$$\text{Direct} = \pm \frac{a}{e} = \pm \frac{2}{\frac{\sqrt{5}}{2}}$$

$$= \pm \sqrt{5} \quad \text{vertices } (0, \pm 2)$$

- (x) Find position vector of a point which divide the join of E with position vector $5\mathbf{i}$ and F with position vector $4\mathbf{i} + \mathbf{j}$ in ratio $2 : 5$.

Ans

$$\begin{aligned} I &= \frac{p\mathbf{b} + q\mathbf{a}}{p + q} \\ &= \frac{5(5\mathbf{i}) + 2(4\mathbf{i} + \mathbf{j})}{5 + 2} \\ &= \frac{33}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} \end{aligned}$$

- (xi) Find direction cosine of $\vec{v} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

Ans

$$\begin{aligned} \underline{v} &= 3\mathbf{i} - \mathbf{j} + 2\mathbf{k} \\ |\underline{v}| &= \sqrt{3^2 + (-1)^2 + 2^2} \\ &= \sqrt{14} \end{aligned}$$

D.Cs are

$$\left[\frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}} \right]$$

- (xii) Find α so that $\vec{u} = \alpha\mathbf{i} + 2\alpha\mathbf{j} - \mathbf{k}$ and $\vec{v} = \mathbf{i} + \alpha\mathbf{j} + 3\mathbf{k}$ are perpendicular.

Ans

$$\underline{u} = \alpha\mathbf{i} + 2\alpha\mathbf{j} - \mathbf{k}$$

$$\underline{v} = \mathbf{i} + \alpha\mathbf{j} + 3\mathbf{k}$$

Since \underline{u} and \underline{v} are perpendicular.

$$\underline{u} \cdot \underline{v} = 0$$

$$(\alpha\mathbf{i} + 2\alpha\mathbf{j} - \mathbf{k})(\mathbf{i} + \alpha\mathbf{j} + 3\mathbf{k}) = 0$$

$$\alpha(1) + 2\alpha(\alpha) - 1(3) = 0$$

$$\alpha + 2\alpha^2 - 3 = 0$$

$$2\alpha^2 + 3\alpha - 2\alpha - 3 = 0$$

$$\alpha(2\alpha + 3) - 1(2\alpha + 3) = 0$$

$$(2\alpha + 3)(\alpha - 1) = 0$$

$$\alpha = 1, \alpha = \frac{-3}{2}$$

(xiii) Prove that if $\underline{a} + \underline{b} + \underline{c} = 0$ then $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$.

Ans Given $\underline{a} + \underline{b} + \underline{c} = 0$

To prove:

$$\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$$

$$\underline{a} + \underline{b} + \underline{c} = 0$$

$$\underline{a} = -(\underline{b} + \underline{c})$$

Take cross product with \underline{b}

$$\underline{a} \times \underline{b} = -(\underline{b} + \underline{c}) \times \underline{b}$$

$$\underline{a} \times \underline{b} = -\underline{b} \times \underline{b} - \underline{c} \times \underline{b}$$

$$= 0 - \underline{c} \times \underline{b}$$

$$= \underline{b} \times \underline{c}$$

Take cross product with \underline{c}

$$\underline{b} = \underline{a} - \underline{c}$$

$$\underline{b} \times \underline{c} = -\underline{a} \times \underline{c} - \underline{c} \times \underline{c}$$

$$= -\underline{a} \times \underline{c} + 0$$

$$= [\underline{c} \times \underline{a}]$$

so the result

$$\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$$

SECTION-II

NOTE: Attempt any Three (3) questions.

$$\text{Q.5.(a)} \text{ If } f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} & x \neq 2 \\ k & x = 2 \end{cases} \quad (5)$$

Find value of k , so that $f(x)$ is continuous at $x = 2$.

$$\begin{aligned} \text{Ans} \rightarrow \lim_{x \rightarrow 2} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} &\times \frac{\sqrt{2x+5} + \sqrt{x+7}}{\sqrt{2x+5} + \sqrt{x+7}} \\ &= \frac{(x-2)}{(x-2)(\sqrt{2x+5} + \sqrt{x+7})} \\ &= \frac{1}{\sqrt{2(2)+5} + \sqrt{2+7}} \\ &= \frac{1}{6} \end{aligned}$$

$$f(2) = k$$

Given $f(x)$ is a continuous function, So

$$f(2) = \lim_{x \rightarrow 2} f(x)$$

$$k = \frac{1}{6}$$

(b) Differentiate $x^2 + \frac{1}{x^2}$ w.r.t. $x - \frac{1}{x}$. (5)

Ans Let $y = x^2 + \frac{1}{x^2}$

$$u = x - \frac{1}{x}$$

$$\frac{dy}{dx} = 2x + \frac{x^2(0) - 1(2x)}{x^4}$$

$$= 2x - \frac{2x}{x^4}$$

$$= 2x - \frac{2}{x^3} = 2\left(x - \frac{1}{x^3}\right) = 2\left(\frac{x^4 - 1}{x^3}\right)$$

$$\frac{dy}{dx} = \frac{2(x^2 - 1)(x^2 + 1)}{x^3}$$

$$\frac{du}{dx} = 1 - \frac{x(0) - 1(1)}{x^2}$$

$$= 1 + \frac{1}{x^2} = \frac{x^2 + 1}{x^2}$$

$$\frac{dy}{du} = \frac{\frac{dy}{dx}}{\frac{du}{dx}} = \frac{2(x^2 - 1)(x^2 + 1)}{x^3} \cdot \frac{x^2}{x^2 + 1}$$

$$= 2\left(x - \frac{1}{x}\right)$$

Q.6.(a) Evaluate $\int \frac{1}{x(x^3 - 1)} dx.$ (5)

Ans $\int \frac{1}{x(x^3 - 1)} dx$

$$\frac{1}{x(x^3 - 1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{Cx + D}{x^2 + x + 1}$$

$$\int \frac{-1}{x} + \frac{1}{x-1} + \frac{2x+1}{x^2+x+1}$$

$$\begin{aligned}
 &= -1 \int (x)^{-1} \cdot 1 dx + \int (x-1)^{-1} \cdot 1 dx + \int (x^2+x+1)^{-1} (2x+1) dx \\
 &= -\ln|x| + \ln|x-1| + \ln|x^2+x+1| + c \\
 &= -1 \ln|x| + \ln|x-1|(x^2+x+1) + c \\
 &= -1 \ln|x| + \ln|x^3-1| + c
 \end{aligned}$$

- (b) Find a joint equation of the straight lines through the origin perpendicular to the lines represented by $x^2 + xy - 6y^2 = 0$. (5)

Ans $x^2 + xy - 6y^2 = 0$
 $(x-2y)(x+3y) = 0$

$$\begin{array}{ll} x-2y=0 & (\text{i}) \\ x+3y=0 & (\text{ii}) \end{array}$$

The line through (0, 0) and \perp (i) is

$$y = -2x \quad \text{or} \quad y + 2x = 0$$

Similarly, the line through (0, 0) and \perp (ii) is

$$y - 3x = 0$$

Joint eq. of line

$$(y + 2x)(y - 3x) = 0$$

$$y^2 - xy - 6x^2 = 0$$

- Q.7.(a) Find the area between x-axis and the curve $y = \sqrt{2ax - x^2}$; $a > 0$. (5)

Ans To find x-intercept

$$y = \sqrt{2ax - x^2}$$

$$\text{If } y = 0; \sqrt{2ax - x^2} = 0 \Rightarrow 2ax - x^2 = 0$$

$$x(2a - x) = 0 \Rightarrow x = 0, x = 2a$$

$$x = 0, 2a$$

$$A = \int_0^{2a} y d(x)$$

$$= \int_0^{2a} \sqrt{2ax - x^2} dx$$

$$= \int_0^{2a} \sqrt{2ax - x^2 - a^2 + a^2} dx \Rightarrow \int_0^{2a} a^2 - (x^2 + a^2 - 2ax) dx$$

$$= \int_0^{2a} \sqrt{a^2 - (x-a)^2} dx$$

Let $x - a = a \sin \theta$ for $\left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right)$

$$\begin{aligned}
 dx &= a \cos \theta d\theta \\
 (x - a)^2 &= a^2 \sin^2 \theta \\
 &= \int_{-\pi/2}^{\pi/2} \sqrt{a^2 - a^2 \sin^2 \theta} a \cos \theta d\theta \\
 &= \int_{-\pi/2}^{\pi/2} \sqrt{a^2 (1 - \sin^2 \theta)} a \cos \theta d\theta \\
 &= \int_{-\pi/2}^{\pi/2} \sqrt{a^2 \cos^2 \theta} \cdot a \cos \theta d\theta \\
 &= \int_{-\pi/2}^{\pi/2} a^2 \cos^2 \theta d\theta \\
 &= \int_{-\pi/2}^{\pi/2} a^2 \left(\frac{1 + \cos 2\theta}{2} \right) d\theta \\
 &= \frac{a^2}{2} \left[\int_{-\pi/2}^{\pi/2} 1 d\theta + \int_{-\pi/2}^{\pi/2} \cos 2\theta d\theta \right] \\
 &= \frac{a^2}{2} \left| \theta + \frac{\sin 2\theta}{2} \right|_{-\pi/2}^{\pi/2} \\
 &= \frac{a^2}{2} \left[\left(\frac{\pi}{2} - \left(-\frac{\pi}{2} \right) \right) + \frac{1}{2} \sin 2\left(\frac{\pi}{2}\right) - \sin 2\left(-\frac{\pi}{2}\right) \right] \\
 &= \frac{a^2}{2} \left[\frac{2\pi}{2} + \frac{1}{2} (\sin \pi - \sin (-\pi)) \right] \\
 &= \frac{a^2}{2} \left[\pi + \frac{1}{2} (0 - 0) \right] \\
 &= \frac{\pi a^2}{2} \text{ sq units.}
 \end{aligned}$$

- (b) Shade the feasible region and also find the corner points of: (5)

$$2x - 3y \leq 6; 2x + 3y \leq 12; x \geq 0; y \geq 0$$

Ans →

$$2x - 3y = 6 \quad (i)$$

$$2x + 3y = 12 \quad (ii)$$

x-intercept

$$y = 0 \Rightarrow x = 3$$

$$P = (3, 0)$$

y-intercept

$$\text{put } x = 0$$

$$y = -2$$

For eq. (i) corner points are $(0, -2)$ $(3, 0)$.

x-intercept

$$x = 0 \Rightarrow y = 4 \quad P(0, 4)$$

y-intercept

$$y = 0 \Rightarrow x = 6 \quad P(6, 0)$$

For eq. (ii) corner points are $(0, 4)$ $(6, 0)$.

Test Point

Put $(0, 0)$ in

$$2x - 3y < 6$$

$0 < 6$ which is true.

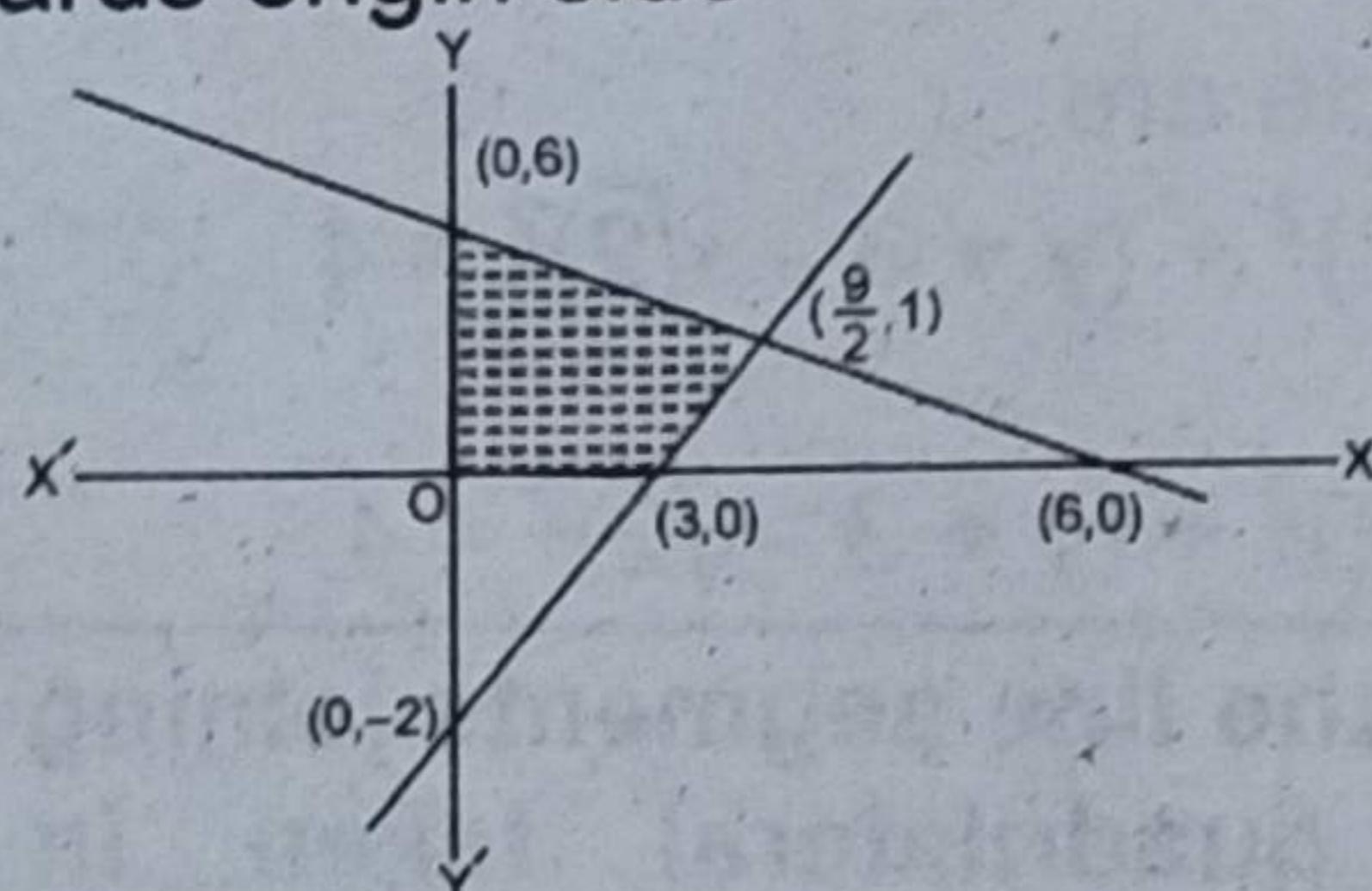
Graph of $2x - 3y \leq 6$ will be towards origin side

Put $(0, 0)$ in

$$2x + 3y < 12$$

$0 < 12$ true

Graph towards origin side



Linear points $(0, 0)$, $(3, 0)$

$$(0, 6), \left(\frac{9}{2}, 1\right)$$

Q.8.(a) Find equation of the circle of radius 2 and tangent to the line $x - y - 4 = 0$ at A $(1, -3)$. (5)

Ans Let (h, k) be the centre of circle

$$r = 2$$

$$\text{eq. of circle } (x - h)^2 + (y - k)^2 = (2)^2$$

$$(1 - h)^2 + (-3 - k)^2 = 4$$

$$h^2 + k^2 - 2h + 6k + 6 = 0 \quad (i)$$

Slope of line $x - y - 4 = 0$ is

$$m_1 = \frac{-1}{-1} = 1$$

Slope of radius

$$m_2 = \frac{k+3}{h-1}$$

$$m_1 m_2 = -1$$

$$(1) \left(\frac{k+3}{h-1} \right) = -1$$

$$k+3 = -1(h-1)$$

$$k+3 = -h+1$$

$$k+h = 1-3$$

$$k+h = -2$$

$$k = -h-2$$

(ii)

Put value of k in eq. (i)

$$h^2 + (-h-2)^2 - 2h + 6(-h-2) + 6$$

$$2h^2 - 4h - 2 = 0 \quad \text{or} \quad h^2 - 2h - 1 = 0$$

By solving equation using quadratic formula

$$h = 1 \pm \sqrt{2}$$

put value of h in eq. (ii)

$$k = -3 \pm \sqrt{2}$$

Eqs. of circle are

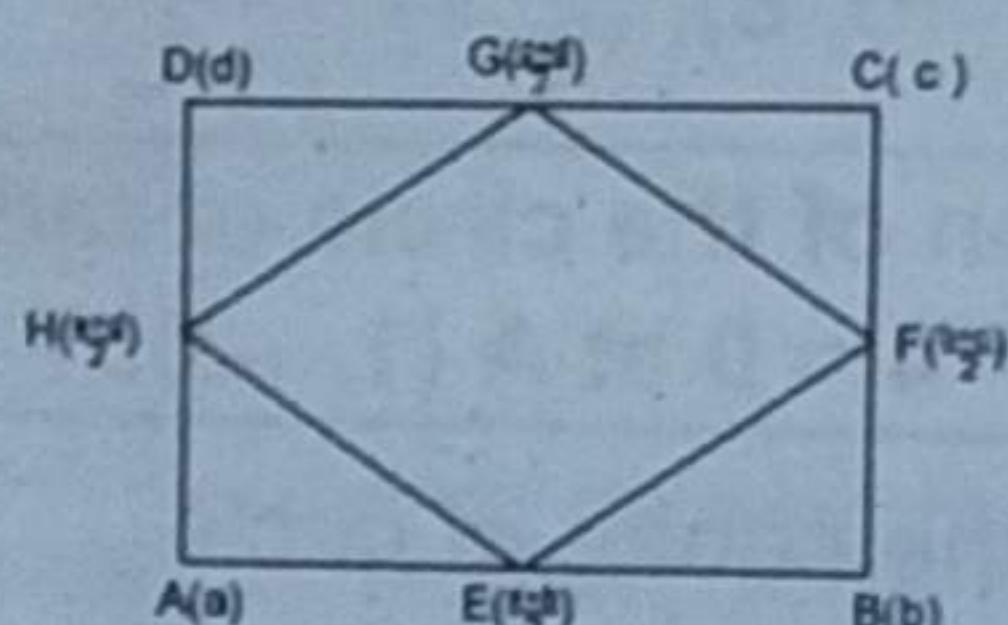
$$(x-1-\sqrt{2})^2 + (y+3-\sqrt{2})^2 = 4$$

and

$$(x-1+\sqrt{2})^2 + (y+3-\sqrt{2})^2 = 4$$

- (b) Prove that the line segments joining the mid-points of sides of quadrilateral taken in order form a parallelogram. (5)

Ans



$$\vec{EF} = \frac{\mathbf{b} + \mathbf{c}}{2} - \frac{\mathbf{a} + \mathbf{b}}{2} = \frac{\mathbf{c} - \mathbf{a}}{2}$$

$$\vec{HG} = \frac{\mathbf{c} - \mathbf{a}}{2}$$

$$\vec{FG} = \frac{\mathbf{d} - \mathbf{b}}{2}$$

$$\vec{EH} = \frac{\mathbf{d} - \mathbf{b}}{2}$$

As the opposite line segments joining the mid-points of sides of quadrilateral are equal so they form a parallelogram.

Q.9.(a) Find the focus, vertex and the directrix of the parabola $x^2 - 4x - 8y + 4 = 0$. (5)

Ans $x^2 - 4x - 8y + 4 = 0$

$$(x - 2)^2 = 8y \quad (i)$$

$$\text{Let } x - 2 = X, y - 0 = Y$$

eq. (i) becomes

$$X^2 = 8Y$$

As standard form is $X^2 = 4aY$

$$4a = 8 \Rightarrow a = 2$$

$$\text{Focus} = (0, a)$$

$$(X, Y) = (0, 2)$$

$$(x - 2, y - 0) = (0, 2)$$

$$x = 2, y = 2$$

focus (2, 2)

Vertex

$$\text{Put } X = 0, Y = 0$$

$$x = 2, y = 0$$

vertex = (2, 0)

Directrix

$$Y = -a$$

y = -2

(b) Find the constant α such that the vectors are coplanar

$$\vec{i} - \vec{j} + \vec{k}, \vec{i} - 2\vec{j} - 3\vec{k} \text{ and } 3\vec{i} - \alpha\vec{j} + 5\vec{k}. \quad (5)$$

Ans $\underline{a} = \vec{i} - \vec{j} + \vec{k}$

$$\underline{b} = \vec{i} - 2\vec{j} - 3\vec{k}$$

$$\underline{c} = 3\vec{i} - \alpha\vec{j} + 5\vec{k}$$

Since vectors are co-planer

$$\begin{vmatrix} 1 & -1 & 1 \\ 1 & -2 & -3 \\ 3 & -\alpha & 5 \end{vmatrix} = 0$$

$$1(-10 - 3\alpha) + 1(5 + 9) + 1(-\alpha + 6) = 0$$

$$-10 - 3\alpha + 14 - \alpha + 6 = 0$$

$$-4\alpha + 10 = 0$$

$$-4\alpha = -10$$

$$4\alpha = 10$$

$\alpha = \frac{5}{2}$