

Inter (Part-I) 2015**Mathematics****(Objective Type)****Time: 30 Minutes****Max. Marks: 20**

Note: Four possible answers, A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1- If n is a negative integer then n is:

- | | |
|----------|-------------------|
| (a) 1 | (b) Not defined ✓ |
| (c) Zero | (d) n |

2- Expansion of $(1 + x)^{-1/4}$ is valid only if:

- | | |
|----------------|-----------------|
| (a) $ x > 1$ | (b) $ x < 1$ ✓ |
| (c) $ x < -1$ | (d) $ x > -1$ |

3- For any triangle ABC, with usual notations r_1 is equal to:

- | | |
|----------------------------|------------------------------|
| (a) $\frac{\Delta}{s - b}$ | (b) $\frac{\Delta}{s - a}$ ✓ |
| (c) $\frac{s - a}{\Delta}$ | (d) $\frac{\Delta}{s - c}$ |

4- One radian is equal to:

- | | |
|----------------------|------------------|
| (a) 57.296° ✓ | (b) 57° |
| (c) 56° | (d) 0175° |

5- Types of rational fractions are:

- | | |
|-------|---------|
| (a) 3 | (b) 2 ✓ |
| (c) 4 | (d) 1 |

6- $\sin 2\alpha$ is equal to:

- | | |
|-----------------------------------|---------------------------|
| (a) $1 - 2 \sin^2 \alpha$ | (b) $2 \cos^2 \alpha - 1$ |
| (c) $2 \sin \alpha \cos \alpha$ ✓ | (d) $\sin \alpha$ |

7- The roots of $x^2 + x - 6 = 0$ are:

- | | |
|-------------|-------------|
| (a) Real ✓ | (b) Equal |
| (c) Complex | (d) Trivial |

8- The power set of the empty set is:

- | | |
|----------------|---------------------|
| (a) Empty set | (b) Non-empty set ✓ |
| (c) Proper set | (d) Improper set |

- 9- Pakistan and India play a hockey match, probability that Pakistan will win:
- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) $\frac{1}{3} \checkmark$ (d) 1
- 10- The middle term in the expansion of $(1 + 2x)^6$ is:
- (a) Third (b) Fourth \checkmark
(c) Fifth (d) Sixth
- 11- If $a_n - a_{n-1} = n + 1$ and $a_4 = 14$, then a_5 is equal to:
- (a) 9 (b) 14
(c) 20 \checkmark (d) 5
- 12- The domain of relation $f = \{(a, 1), (b, 1), (c, 1)\}$ is:
- (a) {a, b, c} \checkmark (b) {a}
(c) {b} (d) {1}
- 13- The sum of roots of equation $ax^2 + bx + c = 0$, $a \neq 0$ is:
- (a) $\frac{b}{a}$ (b) $\frac{c}{a}$
(c) $\frac{-b}{a} \checkmark$ (d) $\frac{-c}{a}$
- 14- Inverse of a square matrix A does not exist if A is:
- (a) Singular \checkmark (b) Non-singular
(c) Unit (d) Diagonal
- 15- $\cos^{-1}(-x)$ is equal to:
- (a) $\cos^{-1}x$ (b) $\pi + \cos^{-1}x$
(c) $\pi - \cos^{-1}x \checkmark$ (d) $\sin^{-1}x$
- 16- The range of $y = \sin x$ is equal to:
- (a) $-1 \leq y \leq 1 \checkmark$ (b) $-1 < y < 1$
(c) $-1 \leq x \leq 1$ (d) $-1 \leq y < 1$
- 17- Solution of equation $\tan x = \frac{1}{\sqrt{3}}$ is:
- (a) I & III quad. \checkmark (b) I & II quad.
(c) II & IV quad. (d) I quad.

Inter (Part-I) 2015

Mathematics

(Subjective Type)

Time: 2.30 Min.

Max. Marks: 80

SECTION-I

2. Write short answers to any EIGHT (8) questions: 16

(I) Write any two properties of inequalities.

Ans 1- Trichotomy Property:

$$\forall a, b \in \mathbb{R} \\ \text{either } a = b \text{ or } a > b \text{ or } a < b$$

2- Transitive Property:

$$\forall a, b, c \in \mathbb{R}$$

$$(i) \quad a > b \wedge b > c \Rightarrow a > c$$

$$(ii) \quad a < b \wedge b < c \Rightarrow a < c$$

(ii) Show that $\forall z \in \mathbb{C}, z^2 + z^{-2}$ is a real number.

Ans $z = x + iy$,

$$(z)^2 = (x + iy)^2$$

$$z^2 = x^2 + (iy)^2 + 2xiy$$

$$z^2 = x^2 + (-1)(y^2) + 2xiy \quad \bar{z}^2 = x^2 + (-1)(y^2) - 2xiy$$

$$z^2 = x^2 - y^2 + 2xiy \quad \bar{z}^2 = x^2 - y^2 - 2xiy$$

$$\bar{z} = x - iy$$

$$(\bar{z})^2 = (x - iy)^2$$

$$\bar{z}^2 = x^2 + (iy)^2 - 2xiy$$

As

$$z^2 + \bar{z}^2 = (x^2 - y^2 + 2xiy) + (x^2 - y^2 - 2xiy) \\ = x^2 - y^2 + 2xiy + x^2 - y^2 - 2xiy$$

$$z^2 + \bar{z}^2 = 2x^2 - 2y^2$$

(iii) Construct the truth table for $(p \wedge \sim p) \rightarrow q$.

Ans

p	q	$\sim p$	$p \wedge \sim p$	$(p \wedge \sim p) \rightarrow q$
T	T	F	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(iv) For $A = \{1, 2, 3, 4\}$, find the relation $\{(x, y) | y = x\}$ in A.

Ans $A \times A = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$

$$R = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$$

(v) Define semi-group.

Ans A non empty set together with a binary operation is a semi-group if

1- S is closed w.r.t binary operation.

2- Associative law holds in S w.r.t binary operation.

(vi) If a, b being elements of a group G , then solve the equation $ax = b$.

Ans $a \in G, a^{-1} \in G$

$$a^{-1}(ax) = a^{-1}b$$

$$(a^{-1}a)x = a^{-1}b \Rightarrow x = a^{-1}b$$

(vii) Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$.

Ans Comparing,

$$x + 3 = y$$

$$x = y - 3 \quad (\text{i})$$

$$3y - 4 = 2x \quad (\text{ii})$$

By putting the value of x in equation (ii),

$$3y - 4 = 2(y - 3)$$

$$3y - 4 = 2y - 6$$

$$3y - 2y = -6 + 4$$

$$\boxed{y = -2}$$

By putting the evaluated value of y in (i),

$$x = y - 3$$

$$x = -2 - 3$$

$$\boxed{x = -5}$$

(viii) Find the inverse of $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$.

Ans $|A| = \begin{vmatrix} 2 & 1 \\ 6 & 3 \end{vmatrix} = 2(3) - 6(1) = 6 - 6 = 0$

A^{-1} does not exist as A is singular

(ix) Without expansion verify that:

$$\begin{vmatrix} 1 & a^2 & \frac{a}{bc} \\ 1 & b^2 & \frac{b}{ca} \\ 1 & c^2 & \frac{c}{ab} \end{vmatrix} = 0$$

Ans Multiply C_3 by abc

$$= \frac{1}{abc} \begin{vmatrix} 1 & a^2 & \frac{a}{bc} \times abc \\ 1 & b^2 & \frac{b}{ac} \times abc \\ 1 & c^2 & \frac{c}{ab} \times abc \end{vmatrix}$$

$$= \frac{1}{abc} \begin{vmatrix} 1 & a^2 & a^2 \\ 1 & b^2 & b^2 \\ 1 & c^2 & c^2 \end{vmatrix}$$

Since all the elements of C_2 and C_3 are identical, so,

$$= \frac{1}{abc} (0)$$

$$= 0$$

$$= \text{R.H.S}$$

(x) **Prove that** $(-1 + \sqrt{-3})^4 + (-1 - \sqrt{-3})^4 = -16$

Ans As we know that,

$$\frac{-1 + \sqrt{-3}}{2} = \omega, \quad \frac{-1 - \sqrt{-3}}{2} = \omega^2$$

$$-1 + \sqrt{-3} = 2\omega, \quad -1 - \sqrt{-3} = 2\omega^2$$

$$\begin{aligned} \text{L.H.S.} &= (-1 + \sqrt{-3})^4 + (-1 - \sqrt{-3})^4 \\ &= (2\omega)^4 + (2\omega^2)^4 \\ &= 16\omega^4 + 16\omega^8 \\ &= 16(\omega + \omega^2) \quad \text{as } \omega^3 = 1 \\ &= 16(-1) \\ &= -16 \\ &= \text{R.H.S} \end{aligned}$$

(xi) **Show that** $(x - 2)$ **is a factor of** $x^4 - 13x^2 + 36$

$$P(x) = x^4 - 13x^2 + 36$$

If $x - 2$ is a factor of $P(x)$

$$\text{Then } P(2) = 0$$

$$P(2) = 2^4 - 13(2)^2 + 36$$

$$P(2) = 16 - 52 + 36 = 0$$

so $x - 2$ is a factor of $P(x)$

(xii) **Show that the roots of** $px^2 - (p - q)x - q = 0$ **are rational.**

Ans For rational roots

$$b^2 - 4ac > 0 \text{ and perfect square}$$

$$\begin{aligned} b^2 - 4ac &= \{(p - q)\}^2 - 4(p)(-q) \\ &= p^2 + q^2 - 2pq + 4pq \end{aligned}$$

$$= p^2 + q^2 + 2pq \\ = (p + q)^2$$

Roots are rational.

3. Write short answers to any EIGHT (8) questions:

16

- (i) Resolve $\frac{7x + 25}{(x + 3)(x + 4)}$ into partial fractions.

Ans
$$\frac{7x + 25}{(x + 3)(x + 4)} = \frac{A}{x + 3} + \frac{B}{x + 4}$$

$$= \frac{4}{x + 3} + \frac{3}{x + 4}$$

- (ii) Find the sequence if $a_n - a_{n-1} = n + 1$; $a_4 = 14$

Ans Put $n = 2, 3, 4$ in

$$a_n - a_{n-1} = n + 1$$

$$\text{writing } a_1 = 2, a_2 = 5, a_3 = 9$$

sequence

$$2, 5, 9, 14, \dots$$

- (iii) If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P show that $b = \frac{2ac}{a+c}$.

Ans $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$2\left(\frac{1}{b}\right) = \frac{1}{a} + \frac{1}{c}$$

$$2\left(\frac{1}{b}\right) = \frac{a+c}{ac}$$

$$\frac{2(ac)}{b} = a + c$$

$$\frac{2(ac)}{a+c} = b$$

So, $b = \frac{2ac}{a+c}$ Proved.

- (iv) Find G.M between $-2i$ and $8i$.

Ans $a = -2i, b = 8i$

$$\text{G.M} = \pm \sqrt{-16i^2} = \pm 4$$

- (v) If $a = 1 - x + x^2 - x^3 + \dots$ $|x| < 1$
 $b = 1 + x + x^2 + x^3 + \dots$ $|x| < 1$

show that $2ab = a + b$.

Ans

$$a = 1 - x + x^2 - x^3 + \dots \quad |x| < 1$$

$$= (1 + x)^{-1}$$

$$b = 1 + x + x^2 + x^3 + \dots \quad |x| < 1$$

$$= (1 - x)^{-1}$$

$$= \frac{1}{1 - x}$$

$$\frac{1}{a} = 1 + x$$

$$\frac{1}{b} = 1 - x$$

$$\frac{1}{a} + \frac{1}{b} = 1 + x + 1 - x = 2$$

$$\frac{a+b}{ab} = 2$$

$$\Rightarrow 2ab = a + b$$

(vi) Find the value of n when ${}^{11}P_n = 11 \cdot 10 \cdot 9$

Ans
$${}^{11}P_n = \frac{11 \cdot 10 \cdot 9 \cdot 8!}{8!}$$

$$\frac{11!}{(11-n)!} = \frac{11!}{8!}$$

$$11 - n = 8 \quad n = 3$$

(vii) Evaluate ${}^{20}C_{17}$

Ans
$$\begin{aligned} {}^{20}C_{17} &= \frac{20!}{17! \cdot 3!} \\ &= \frac{20 \cdot 19 \cdot 18 \cdot 17!}{17! \cdot 3 \cdot 2 \cdot 1} \\ &= \frac{20 \cdot 19 \cdot 18}{6} = 1140 \end{aligned}$$

(viii) A die is rolled. What is the probability that dots on the top are greater than 4?

Ans $n(S) = \{1, 2, 3, 4, 5, 6\}$

$$n(A) = \{5, 6\}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

(ix) Find the number of diagonals of a 6-sided figure.

Ans Number of diagonals

$${}^6C_2 - 6$$

$$= 15 - 6 = 9$$

(x) **Prove that $n! > 2^n - 1$ for $n = 4, 5$.**

Ans $n! > 2^n - 1$

For $n = 4$

$$4! > 2^4 - 1$$

$$24 > 15$$

For $n = 5$

$$5! > 2^5 - 1$$

$$120 > 31$$

(xi) **If x is so small that its square and higher powers neglected, then show that $\frac{1-x}{\sqrt{1+x}} \approx 1 - \frac{3}{2}x$.**

Ans L.H.S. $= \frac{1-x}{\sqrt{1+x}}$

$$= (1-x)(1+x)^{-1/2}$$

$$= (1-x)\left(1 - \frac{1}{2}x + \frac{3}{8}x^2 \dots \dots \right)$$

$$= (1-x)\left(1 - \frac{x}{2}\right) \text{ by given condition}$$

$$= 1 - x - \frac{x}{2} + \frac{x^2}{2}$$

$$\approx 1 - \frac{3}{2}x = \text{R.H.S}$$

(xii) **Expand $(1+2x)^{-1}$ up to four terms.**

Ans $(1+2x)^{-1}$

$$= 1 + (-1)(2x) + \frac{(-1)(-2)}{2!} 4x^2 + \frac{(-1)(-2)(-3)}{3!} 8x^3 + \dots$$

$$= 1 - 2x + 4x^2 - 8x^3 + \dots$$

4. Write short answers to any NINE (9) questions: 18

(i) **Define radian.**

Ans Radian is the measure of the angle subtended at the center of the circle by an arc, whose length is equal to the radius of the circle.

(ii) **Prove that $\sec \theta \cosec \theta \sin \theta \cos \theta = 1$**

Ans $\sec \theta \cosec \theta \sin \theta \cos \theta$

$$\frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} \cdot \sin \theta \cdot \cos \theta$$

$$= 1 \quad \text{R.H.S}$$

(iii) **Prove that $\sin 75^\circ$. (Without using calculator)**

Ans $\sin 75^\circ = \sin (45^\circ + 30^\circ)$
 $= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$
 $= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$
 $= \frac{\sqrt{3} + 1}{2\sqrt{2}}$

(iv) Prove that $\frac{\sin A + \sin 2A}{1 + \cos A + \cos 2A} = \tan A.$

Ans L.H.S

$$\begin{aligned} &= \frac{\sin A + \sin 2A}{1 + \cos 2A + \cos A} \\ &= \frac{\sin A + 2 \sin A \cos A}{2 \cos^2 A + \cos A} \\ &= \frac{\sin A (1 + 2 \cos A)}{\cos A (1 + 2 \cos A)} \\ &= \tan A \end{aligned}$$

(v) Find the value of $\cos 2\alpha$, if $\cos \alpha = \frac{3}{5}$, $0 < \alpha < \frac{\pi}{2}$.

Ans $\cos \alpha = \frac{3}{5} \quad 0 < \alpha < \frac{\pi}{2}$

$$\begin{aligned} \cos 2\alpha &= 2 \cos^2 \alpha - 1 \\ &= 2 \left(\frac{9}{25} \right) - 1 = \frac{-7}{25} \end{aligned}$$

(vi) Prove that $\frac{\sin 3\theta}{\cos \theta} + \frac{\cos 3\theta}{\sin \theta} = 2 \cot 2\theta.$

Ans
$$\begin{aligned} &\frac{\sin 3\theta}{\cos \theta} + \frac{\cos 3\theta}{\sin \theta} \\ &= \frac{\sin 3\theta \sin \theta + \cos 3\theta \cdot \cos \theta}{\cos \theta \sin \theta} \\ &= \frac{\cos (3\theta - \theta)}{\sin \theta \cdot \cos \theta} \\ &= \frac{2 \cdot \cos (2\theta)}{2 \cdot \sin \theta \cdot \cos \theta} = \frac{2 \cos 2\theta}{\sin 2\theta} = 2 \cot 2\theta \end{aligned}$$

(vii) Find the period of $3 \cos \frac{x}{5}$.

Ans Period of cosine is 2π

$$3 \cos \frac{x}{5} = 3 \cos \left(\frac{x}{5} + 2\pi \right)$$

$$3 \cos \frac{x}{5} = 3 \cos \frac{1}{5}(x + 10\pi)$$

Period of $3 \cos \frac{x}{5}$ is 10π

- (viii) Find the area of the triangle ABC, if $b = 37$, $c = 45$, $\alpha = 30^\circ 50'$.

Ans $b = 37$, $c = 45$, $\alpha = 30^\circ 50'$

$$\Delta = \frac{1}{2} b \cdot c \sin \alpha$$

$$= \frac{1}{2} \times 37 \times 45 \sin 30^\circ 50'$$

$$= 426.69 \text{ sq. units}$$

- (ix) Prove that $r r_1 r_2 r_3 = \Delta^2$.

Ans L.H.S

$$\begin{aligned} r \cdot r_1 \cdot r_2 \cdot r_3 &= \frac{\Delta}{s} \cdot \frac{\Delta}{s-a} \cdot \frac{\Delta}{s-b} \cdot \frac{\Delta}{s-c} \\ &= \frac{\Delta^4}{\Delta^2} = \Delta^2 \end{aligned}$$

- (x) Define angle of elevation.

Ans While looking any object above the horizontal ray, the angle which the line of sight makes with the horizontal ray is called angle of elevation.

- (xi) Prove that $\tan^{-1} \frac{120}{119} = 2 \cos^{-1} \frac{12}{13}$.

$$\tan^{-1} \frac{120}{119} = 2 \cos^{-1} \frac{12}{13}$$

$$\text{Let, } \cos^{-1} \frac{12}{13} = \theta$$

$$\cos \theta = \frac{12}{13}$$

$$\sin \theta = \sqrt{1 - \cos^2 \theta}$$

$$= \sqrt{1 - \left(\frac{12}{13}\right)^2}$$

$$= \sqrt{1 - \frac{144}{169}}$$

$$= \sqrt{\frac{169 - 144}{169}}$$

$$= \sqrt{\frac{25}{169}}$$

$$\sin \theta = \frac{5}{13}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$= \frac{5}{13} \div \frac{12}{13}$$

$$= \frac{5}{13} \times \frac{13}{12}$$

$$\tan \theta = \frac{5}{12}$$

$$\tan^{-1} \frac{120}{119} = 2 \cos^{-1} \frac{12}{13}$$

$$\tan^{-1} \frac{120}{119} = 2\theta$$

$$\Rightarrow \tan 2\theta = \frac{120}{119}$$

Now,

$$\text{L.H.S} = \tan 2\theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \times \frac{5}{12}}{1 - \frac{25}{144}}$$

$$= \frac{10}{12} \times \frac{144}{119} = \frac{120}{119}$$

$$= \text{R.H.S}$$

Thus proved that

$$\tan^{-1} \frac{120}{119} = 2 \cos^{-1} \frac{12}{13}$$

(xii) Find the solution of $2 \sin^2 \theta - \sin \theta = 0$.

Ans $2 \sin^2 \theta - \sin \theta = 0$

$$\sin \theta (2 \sin \theta - 1) = 0$$

$$\sin \theta = 0 \quad \sin \theta = \frac{1}{2}$$

$$\Rightarrow 0 = n\pi \quad n \in \mathbb{Z}$$

$$\sin 0 = \frac{1}{2}$$

$$0 = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\left\{ \frac{\pi}{6} + 2n\pi \right\}, \left\{ \frac{5\pi}{6} + 2n\pi \right\}$$

solution

$$\{n\pi\} \cup \left\{ \frac{\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{6} + 2n\pi \right\}$$

(xiii) Find the solution $\cot \theta = \frac{1}{\sqrt{3}}$ which lie in $[0, 2\pi]$.

Ans $\cot \theta = \frac{1}{\sqrt{3}}$

$$\tan \theta = \sqrt{3}$$

θ is in 1st and 3rd quadrant

$$\theta = \frac{\pi}{3}, 4 \frac{\pi}{3}$$

SECTION-II

NOTE: Attempt any THREE (3) questions.

Q.5.(a) Solve the system of linear equation by Cramer's

rule	$2x_1 - x_2 + x_3 = 5$ $4x_1 + 2x_2 + 3x_3 = 8$ $3x_1 - 4x_2 - x_3 = 3$	(5)
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Ans

$$2x_1 - x_2 + x_3 = 5$$

$$4x_1 + 2x_2 + 3x_3 = 8$$

$$3x_1 - 4x_2 - x_3 = 3$$

$$AX = B$$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 4 & 2 & 3 \\ 3 & -4 & -1 \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, B = \begin{bmatrix} 5 \\ 8 \\ 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 & 1 \\ 4 & 2 & 3 \\ 3 & -4 & -1 \end{vmatrix}$$

$$\begin{aligned}
 &= 2 \begin{vmatrix} 2 & 3 \\ -4 & -1 \end{vmatrix} - (-1) \begin{vmatrix} 4 & 3 \\ 3 & -1 \end{vmatrix} + 1 \begin{vmatrix} 4 & 2 \\ 3 & -4 \end{vmatrix} \\
 &= 2[(2)(-1) - (-4)(3)] + 1[(4)(-1) - (3)(3)] \\
 &\quad + 1[(4)(-4) - (3)(2)] \\
 &= 2(-2 + 12) + 1(-4 - 9) + 1(-16 - 6) \\
 &= 2(10) + 1(-13) + 1(-22) \\
 &= 20 - 13 - 22
 \end{aligned}$$

$$|A| = -15$$

$$x_1 = \frac{\begin{vmatrix} 5 & -1 & 1 \\ 8 & 2 & 3 \\ 3 & -4 & -1 \end{vmatrix}}{|A|}$$

$$x_1 = \frac{5[(2)(-1) - (-4)(3)] - (-1)[(8)(-1) - (3)(3)] + 1[(8)(-4) - (3)(2)]}{-15}$$

$$x_1 = \frac{5(-2 + 12) + 1(-8 - 9) + 1(-32 - 6)}{-15}$$

$$x_1 = \frac{5(10) + 1(-17) + 1(-38)}{-15}$$

$$x_1 = \frac{50 - 17 - 38}{-15}$$

$$x_1 = \frac{-5}{-15}$$

$$x_1 = \frac{1}{3}$$

$$x_2 = \frac{\begin{vmatrix} 2 & 5 & 1 \\ 4 & 8 & 3 \\ 3 & 3 & -1 \end{vmatrix}}{|A|}$$

$$= \frac{2[(8)(-1) - (3)(3)] - 5[(4)(-1) - (3)(3)] + 1[(4)(3) - (3)(8)]}{-15}$$

$$= \frac{2(-8 - 9) - 5(-4 - 9) + 1(12 - 24)}{-15}$$

$$= \frac{2(-17) - 5(-13) + 1(-12)}{-15}$$

$$= \frac{-34 + 65 - 12}{-15}$$

$$\begin{aligned}
 &= \frac{19}{-15} \\
 x_2 &= -\frac{19}{15} \\
 x_3 &= \frac{\begin{vmatrix} 2 & -1 & 5 \\ 4 & 2 & 8 \\ 3 & -4 & 3 \end{vmatrix}}{|A|} \\
 &= \frac{2[(2)(3) - (-4)(8)] - (-1)[(4)(3) - (3)(8)] + 5[(4)(-4) - (3)(2)]}{-15} \\
 &= \frac{2(6 + 32) + 1(12 - 24) + 5(-16 - 6)}{-15} \\
 &= \frac{2(38) + 1(-12) + 5(-22)}{-15} \\
 &= \frac{76 - 12 - 110}{-15} = \frac{-46}{-15} \\
 x_3 &= \frac{46}{15}
 \end{aligned}$$

(b) Solve the equation $4^x - 3 \cdot 2^{x+3} + 128 = 0$.

Ans $4^x - 3 \cdot 2^{x+3} + 128 = 0$

$$(2^x)^2 - 24(2^x) + 128 = 0$$

$$2^x = y$$

$$y^2 - 24y + 128 = 0$$

$$y^2 - 16y - 8y + 128 = 0$$

$$y(y - 16) - 8(y - 16) = 0$$

$$(y - 16)(y - 8) = 0$$

$$y = 16, 8$$

$$2^x = 16$$

$$2^x = 2^4$$

By comparing, we get

$$x = 4$$

Similarly,

$$2^x = 8$$

$$2^x = 2^3$$

By comparing, we get

$$x = 3$$

$$\text{S.S} = \{4, 5\}$$

Q.6.(a) Resolve $\frac{1}{(x-3)^2(x+1)}$ into partial fractions. (5)

Ans $\frac{1}{(x-3)^2(x+1)} = \frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

$$1 = A(x-3)^2 + B(x-3)(x+1) + C(x+1)$$

Multiplying throughout by $(x-3)^2(x+1)$

$$1 = A(x-3)^2 + B(x-3)(x+1) + C(x+1)$$

Put $x+1=0 \Rightarrow x=-1$ in eq (i)

$$1 = A(-1-3)^2 + B(-1-3)(-1+1) + C(-1+1)$$

$$1 = A(-4)^2 + B(0) + C(0)$$

$$\boxed{\frac{1}{16} = A}$$

Put $x-3=0 \Rightarrow x=3$ in eq. (i)

$$1 = A(3-3)^2 + B(3-3)(3+1) + C(3+1)$$

$$1 = A(0) + B(0) + C(4)$$

$$\boxed{\frac{1}{4} = C}$$

Comparing coefficients of x^2 , x and constant from eq (i)

$$A+B=0$$

$$B=-A$$

$$\boxed{B = \frac{-1}{16}}$$

$$\frac{1}{(x-3)^2(x+1)} = \frac{1}{16(x+1)} + \frac{-1}{16(x-3)} + \frac{1}{4(x-3)^2}$$

(b) For what value of n , $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the positive geometric mean between 'a' and 'b'? (5)

Ans $\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \sqrt{ab}$

$$a^n + b^n = a^{n-\frac{1}{2}} b^{\frac{1}{2}} + a^{\frac{1}{2}} b^{n-\frac{1}{2}}$$

$$a^n - a^{n-\frac{1}{2}} b^{\frac{1}{2}} = a^{\frac{1}{2}} b^{n-\frac{1}{2}} - b^n$$

$$a^{n-\frac{1}{2}} (a^{\frac{1}{2}} - b^{\frac{1}{2}}) = b^{n-\frac{1}{2}} (a^{\frac{1}{2}} - b^{\frac{1}{2}})$$

$$a^{n-\frac{1}{2}} = b^{n-\frac{1}{2}}$$

$$\left(\frac{a}{b}\right)^{n-\frac{1}{2}} = 1 = \left(\frac{a}{b}\right)^0$$

$$n - \frac{1}{2} = 0 \Rightarrow n = \frac{1}{2}$$

Q.7.(a) How many 6-digit numbers can be formed without repeating any digit from the digits 0, 1, 2, 3, 4, 5? In how many of them will 0 be at the tens place? (5)

Ans Numbers using given six digits

$$= {}^6P_6 = 720$$

When 0 is at first place then number are $= {}^5P_5$
= 120

Required 6-digits number

$$720 - 120 = 600$$

Numbers when 0 is at tens place

$$= {}^5P_5 = 5! = 120$$

(b) If x is very nearly equal 1, then prove that $px^p - qx^q \approx (p - q)x^{p+q}$. (5)

Ans $px^p - qx^q \approx (p - q)x^{p+q}$

L.H.S

$$px^p - qx^q$$

Let $x = 1 + h$ where

h is so small that its square and higher powers may be neglected

$$\begin{aligned} & p(1 + h)^p - q(1 + h)^q \\ &= p(1 + ph + \dots) - q(1 + qh + \dots) \\ &= p(1 + ph) - q(1 + qh) \end{aligned}$$

Neglecting square and higher powers of h

$$\begin{aligned} &= p + p^2h - q - q^2h \\ &= (p - q) + (p^2 - q^2)h \\ &= (p - q)\{1 + (p + q)h\} \end{aligned}$$

R.H.S

$$(p - q)x^{p+q}$$

$$\text{put } x = 1 + h$$

$$(p - q) x^{p+q}$$

$$= (p - q) (1 + h)^{p+q}$$

$$= (p - q) \{1 + (p + q) h + \dots\}$$

Neglecting h^2 and higher powers of h

$$= (p - q) \{1 + (p + q) h\}$$

from (1) and (2)

L.H.S \approx R.H.S

Q.8.(a) Prove the identity: $\sin^6 \theta + \cos^6 \theta = 1 - 3 \sin^2 \theta \cos^2 \theta.$ (5)

Ans $\sin^6 \theta + \cos^6 \theta = 1 - 3 \sin^2 \theta \cos^2 \theta$

L.H.S

$$\sin^6 \theta + \cos^6 \theta = (\sin^2 \theta)^3 + (\cos^2 \theta)^3$$

$$= (\sin^2 \theta + \cos^2 \theta) \{(\sin^2 \theta)^2 + (\cos^2 \theta)^2 - \sin^2 \theta \cos^2 \theta\}$$

$$= \sin^4 \theta + \cos^4 \theta - \sin^2 \theta \cos^2 \theta$$

$$= \sin^4 \theta + \cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta - 3 \sin^2 \theta \cos^2 \theta$$

$$= (\sin^2 \theta + \cos^2 \theta)^2 - 3 \sin^2 \theta \cos^2 \theta$$

$$= 1 - 3 \sin^2 \theta \cos^2 \theta$$

= R.H.S

(b) Prove the identity $\sqrt{\frac{1 + \sin \alpha}{1 - \sin \alpha}} = \frac{\sin \frac{\alpha}{2} + \cos \frac{\alpha}{2}}{\sin \frac{\alpha}{2} - \cos \frac{\alpha}{2}}$ (5)

Ans $\sqrt{\frac{1 + \sin \alpha}{1 - \sin \alpha}} = \frac{\sin \frac{\alpha}{2} + \cos \frac{\alpha}{2}}{\sin \frac{\alpha}{2} - \cos \frac{\alpha}{2}}$

$$\cos^2 \frac{\alpha}{2} + \sin^2 \frac{\alpha}{2} = 1$$

$$\sin \alpha = 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}$$

L.H.S

$$\begin{aligned}
 &= \sqrt{\frac{\cos^2 \frac{\alpha}{2} + \sin^2 \frac{\alpha}{2} + 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}}{\cos^2 \frac{\alpha}{2} + \sin^2 \frac{\alpha}{2} - 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}}} \\
 &= \sqrt{\frac{(\cos \frac{\alpha}{2} + \sin \frac{\alpha}{2})^2}{(\sin \frac{\alpha}{2} - \cos \frac{\alpha}{2})^2}} \\
 &= \frac{\sin \frac{\alpha}{2} + \cos \frac{\alpha}{2}}{\sin \frac{\alpha}{2} - \cos \frac{\alpha}{2}} = \text{R.H.S}
 \end{aligned}$$

Q.9.(a) Solve the triangle ABC in which $a = 7$, $b = 3$, $\gamma = 38^\circ 13'$. (5)

Ans $a = 7$, $b = 3$ $\gamma = 38^\circ 13'$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$c = 5$$

$$\begin{aligned}
 c^2 &= (7)^2 + (3)^2 - 2(7)(3) \cos (38^\circ 13') \\
 &= 49 + 9 - 42 \cos (38.22)^\circ
 \end{aligned}$$

$$c^2 = 25 \Rightarrow c = 5$$

$$\frac{c}{\sin \gamma} = \frac{b}{\sin \beta}$$

$$\frac{5}{\sin (38.22)^\circ} = \frac{3}{\sin \beta}$$

$$\frac{5}{0.619} = \frac{3}{\sin \beta}$$

$$\sin \beta = \frac{3(0.619)}{5}$$

$$\begin{aligned}
 \beta &= \sin^{-1} (0.371) \\
 &= 21.79^\circ
 \end{aligned}$$

$$= 21 + \left[\frac{.79}{100} \times 60 \right]$$

$$\boxed{\beta = 21^\circ 47'}$$

Now,

$$\alpha + \beta + \gamma = 180^\circ$$

$$\begin{aligned}\alpha &= \beta + \gamma - 180^\circ \\ &= 21^\circ 47' + 38^\circ 13' - 180^\circ \\ &= 21.79^\circ + 38.22^\circ - 180^\circ\end{aligned}$$

$$\boxed{\alpha = 120^\circ}$$

(b) Prove that $2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} = \frac{\pi}{4}$. (5)

Ans $\rightarrow 2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} = \frac{\pi}{4}$

$$\begin{aligned}2 \tan^{-1} \frac{1}{3} &= \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{3} \\ &= \tan^{-1} \frac{3}{4}\end{aligned}$$

L.H.S

$$\begin{aligned}2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} \\ &= \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{7} \\ &= \tan^{-1} (1) \\ &= \frac{\pi}{4} = \text{R.H.S}\end{aligned}$$