Inter (Part-II) 2018

Mathematics	Group-II	PAPER: II
Time: 30 Minutes	(OBJECTIVE TYPE)	Marks: 20

Note: Four possible answers, A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1- $\frac{d}{dx}\cos h\, x =$:

(a) $-\sin hx$ (b) $\sec hx$
(c) $-\sec hx$ (d) $\sin hx$ ✓

2- **Solution of $\frac{dy}{dx} = \frac{-y}{x}$ is:**

(a) $\frac{x}{y} = c$ (b) $\frac{y}{x} = c$
(c) $y = cx$ (d) $xy = c$ ✓

3- **If at least one vertical line meets the curve at more than two points, then curve is:**

(a) A function (b) Not a function ✓
(c) One-to-one function (d) Onto function

4- $\int \sec^2 x\, dx$:

(a) $\tan x$ ✓ (b) $\frac{\sec^3 x}{3}$
(c) $\tan^2 x$ (d) $\sec x \tan x$

5- **Domain of $f(x) = x^2 + 1$:**

(a) R ✓ (b) $R - \{1\}$
(c) $R - \{-1\}$ (d) $[1, \infty]$

6- $\int \sin x \cos x\, dx$:

(a) $\frac{1}{2}\cos 2x$ (b) $-\frac{1}{2}\cos 2x$
(c) $\frac{\sin^2 x}{2}$ ✓ (d) $\frac{\cos^2 x}{2}$

7- If $x = f(\theta)$, $y = g(\theta)$, then $\frac{dy}{dx}$:

(a) $\frac{dy}{d\theta}\frac{d\theta}{dx}$ √ (b) $\frac{dx}{d\theta}\frac{d\theta}{dy}$

(c) $\frac{d\theta}{dy}\frac{dx}{d\theta}$ (d) $\frac{dy}{d\theta}\frac{dx}{d\theta}$

8- $\frac{d}{dx}\log_a x =$:

(a) $\frac{1}{x}$ (b) $x\ln x - x$

(c) $\frac{1}{x}\ln a$ (d) $\frac{1}{x\ln a}$ √

9- $\int \frac{1}{x\sqrt{x^2-1}}dx$:

(a) $\sin^{-1} x$ (b) $\tan^{-1} x$

(c) $\sec^{-1} x$ √ (d) $\operatorname{cosec}^{-1} x$

10- $\frac{d}{dx}\sec hx =$:

(a) $\sec hx \tan h x$ (b) $-\sec hx \tan h x$ √

(c) $\tan h^2 x$ (d) $\sec h^2 x$

11- For ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $(a > b)$, then eccentricity $e =$:

(a) $\frac{\sqrt{a^2-b^2}}{a}$ √ (b) $\frac{\sqrt{a^2+b^2}}{a}$

(c) $\frac{\sqrt{b^2-a^2}}{a}$ (d) $\frac{\sqrt{b^2-a^2}}{b}$

12- Horizontal line through $(7, -9)$ is:

(a) $x = 7$ (b) $x = -9$

(c) $y = 7$ (d) $y = -9$ √

13- Projection of vector $\vec{u}$ on vector $\vec{v}$ is:

(a) $\frac{\vec{u}\cdot\vec{v}}{|v|}$ √ (b) $\frac{\vec{u}\cdot\vec{v}}{|u|}$

(c) $\frac{\vec{u}\times\vec{v}}{|v|}$ (d) $\frac{\vec{u}\times\vec{v}}{|u|}$

14- **System of linear inequalities involved in the problem is called:**

(a) Coefficients (b) Solution
(c) Problem constraints ✓
(d) Boundaries

15- **If $\vec{v}$ is any vector, then vector of magnitude 5 opposite to $\vec{v}$ is:**

(a) $5\vec{v}$ (b) $-5\vec{v}$
(c) $5\frac{\vec{v}}{|v|}$ (d) $-5\frac{\vec{v}}{|v|}$ ✓

16- **Equation of line bisecting II and IV quadrant:**

(a) $y = x$ (b) $y = -x$ ✓
(c) $y = \frac{1}{x}$ (d) $x + y = 1$

17- **Joint equation of two lines is $ax^2 + 2hxy + by^2 = 0$, if θ is angle between them, then $\tan\theta =$:**

(a) $\frac{2\sqrt{h^2 + ab}}{a + b}$ (b) $\frac{2\sqrt{h^2 - ab}}{a + b}$ ✓
(c) $\frac{\sqrt{h^2 + ab}}{a + b}$ (d) $\frac{\sqrt{h^2 - ab}}{a + b}$

18- **Set of all points equidistant from a fixed point form:**

(a) Ellipse (b) Parabola
(c) Hyperbola (d) Circle ✓

19- **Distance of (x_1, y_1) from line $ax + by + c = 0$ is:**

(a) $\frac{|ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$ ✓ (b) $\frac{|ax_1 + by_1 - c|}{\sqrt{a^2 + b^2}}$
(c) $\frac{|ax_1 + by_1 + c|}{\sqrt{a + b}}$ (d) $\frac{|ax_1 + by_1 - c|}{\sqrt{a + b}}$

20- **Focal chord perpendicular to axis of parabola is called:**

(a) Latus Rectum ✓ (b) Eccentricity
(c) Vertex (d) Axis

Inter (Part-II) 2018

Mathematics	Group-II	PAPER: II
Time: 2.30 Hours	(SUBJECTIVE TYPE)	Marks: 80

SECTION-I

2. Write short answers to any EIGHT (8) questions: 16

(i) Prove that $\cos h^2 x + \sin h^2 x = \cos h\, 2x$.

Ans L.H.S $= \cos h^2 x + \sin h^2 x$

$$= \left(\frac{e^x + e^{-x}}{2}\right)^2 \left(\frac{e^x - e^{-x}}{2}\right)^2$$

$$= \frac{e^{2x} + e^{-2x} + 2e^x e^{-x}}{4} + \frac{e^{2x} + e^{-2x} - 2e^x e^{-x}}{4}$$

$$= \frac{e^{2x} + e^{-2x} + \cancel{2e^x e^{-x}} + e^{2x} + e^{-2x} - \cancel{2e^x e^{-x}}}{4}$$

$$= \frac{2e^{2x} + 2e^{-2x}}{4}$$

$$= \frac{{}^1\cancel{2}(e^{2x} + e^{-2x})}{\cancel{4}_2}$$

$$= \frac{e^{2x} + e^{-2x}}{2}$$

$$= \cos h\, 2x$$

Hence Proved.

L.H.S = R.H.S.

(ii) Determine whether function $f(x) = \dfrac{x^3 - x}{x^2 + 1}$ is even or odd.

Ans Let $f(x) = f(-x)$

So, $$f(-x) = \frac{(-x)^3 - (-x)}{(-x)^2 + 1}$$

$$= \frac{-x^3 + x}{x^2 + 1}$$

$$= -\left[\frac{(x^3 - x)}{x^2 + 1}\right]$$

$$f(-x) = -f(x)$$

So $f(x)$ is an odd function.

(iii) Evaluate $\lim\limits_{x \to 0} \dfrac{\sec x - \cos x}{x}$.

Ans $\lim_{x\to 0} \frac{\sec x - \cos x}{x}$

$$= \lim_{x\to 0}\left(\frac{\frac{1}{\cos x} - \cos x}{x}\right) \qquad \left(\because \ \sec x = \frac{1}{\cos x}\right)$$

$$= \lim_{x\to 0} \frac{1-\cos^2 x}{x\cos x}$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x\cos x}$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x\cos x} \times \frac{x}{x}$$

$$= \lim_{x\to 0}\left(\frac{\sin^2 x}{x^2} \times \frac{x}{\cos x}\right)$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x^2} \times \lim_{x\to 0} \frac{x}{\cos x}$$

$$(1)^2 \times 0 = 0$$

(iv) **Find $\frac{dy}{dx}$ if $y = \frac{a+x}{a-x}$.**

Ans As $y = \frac{a+x}{a-x}$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{a+x}{a-x}\right]$$

$$= \frac{(a-x)\frac{d}{dx}(a+x) - (a+x)\frac{d}{dx}(a-x)}{(a-x)^2}$$

$$= \frac{(a-x)(1) - (a+x)(-1)}{(a-x)^2}$$

$$= \frac{a - \cancel{x} + a + \cancel{x}}{(a-x)^2}$$

$$= \frac{2a}{(a-x)^2}$$

(v) **Find $\frac{dy}{dx}$ if $x^2 - 4xy - 5y = 0$.**

Ans $x^2 - 4xy - 5y = 0$

$$\frac{d}{dx}(x^2 - 4xy - 5y) = \frac{d}{dx}(0)$$

$$\frac{d}{dx}(x^2) - \frac{d}{dx}(4xy) - \frac{d}{dx}(5y) = 0$$

$$\frac{d}{dx}(x^2) - 4\frac{d}{dx}(xy) - 5\frac{d}{dx}(y) = 0$$

$$2x - 4\left[x \cdot \frac{dy}{dx} + y \cdot 1\right] - 5\frac{dy}{dx} = 0$$

$$2x - 4x\frac{dy}{dx} - 4y - 5\frac{dy}{dx} = 0$$

$$4x\frac{dy}{dx} + 5\frac{dy}{dx} = 2x - 4y$$

$$\frac{dy}{dx}(4x + 5) = 2(x - 2y)$$

$$\frac{dy}{dx} = \frac{2(x - 2y)}{4x + 5}$$

(vi) Differentiate $x^2 - \frac{1}{x^2}$ w.r.t x^4.

Ans Let $y = x^2 - \frac{1}{x^2}$; $u = x^4$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 - \frac{1}{x^2}\right)$$

$$= \frac{d}{dx}(x^2) - \frac{d}{dx}\left(\frac{1}{x^2}\right)$$

$$= 2x - \left[\frac{x^2(0) - 1(2x)}{x^4}\right]$$

$$= 2x - \frac{0 - 2x}{x^4}$$

$$= 2x + \frac{2x}{x^4}$$

$$= 2x + \frac{2}{x^3}$$

$$= \frac{2x^4 + 2}{x^3}$$

$$\frac{dy}{dx} = \frac{2(x^4 + 1)}{x^3}$$

As $u = x^4$

$$\frac{du}{dx} = 4x^3$$

$$\frac{dy}{du} = \frac{dy}{dx} \times \frac{dx}{du}$$

$$= \frac{2(x^4+1)}{4x^3 \cdot x^3}$$

$$= \frac{x^4+1}{2x^6}$$

(vii) **Differentiate $\sin^{-1}\sqrt{1-x^2}$ w.r.t 'x'.**

Ans Let $y = \sin^{-1}\sqrt{1-x^2}$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin^{-1}\sqrt{1-x^2})$$

$$= \frac{1}{\sqrt{1-(\sqrt{1-x^2})^2}} \cdot \frac{d}{dx}\sqrt{1-x^2}$$

$$= \frac{1}{\sqrt{1-1+x^2}}\left[\frac{1}{2}(1-x^2)^{-1/2}(-2x)\right]$$

$$= \frac{1}{\sqrt{x^2}} \cdot \frac{-x}{\sqrt{1-x^2}} = \frac{1}{x} \cdot \frac{-x}{\sqrt{1-x^2}}$$

$$= \frac{-1}{\sqrt{1-x^2}}$$

(viii) **Find $\frac{dy}{dx}$ if $y = \ln\left(x+\sqrt{x^2+1}\right)$.**

Ans

$$\frac{dy}{dx} = \frac{d}{dx}\left[\ln(x+\sqrt{x^2+1})\right]$$

$$= \frac{1}{x+\sqrt{x^2+1}} \frac{d}{dx}\left(x+\sqrt{x^2+1}\right)$$

$$= \frac{1}{x+\sqrt{x^2+1}} \cdot \left[1+\frac{1}{2}(x^2+1)^{-1/2}(2x)\right]$$

$$= \frac{1}{x+\sqrt{x^2+1}} \cdot \left[1+\frac{1x}{\sqrt{x^2+1}}\right]$$

$$= \frac{1}{\sqrt{x^2+1}}$$

(ix) **Find $\frac{dy}{dx}$ if $y = e^{-2x}\sin 2x$.**

Ans

$$\frac{dy}{dx} = \frac{d}{dx}(e^{-2x}\sin 2x)$$

$$= e^{-2x}(\sin 2x)' + \sin 2x\,(e^{-2x})'$$

$$= e^{-2x} . \cos 2x\,(2) + \sin 2x . e^{-2x}\,(-2)$$
$$= 2e^{-2x} \cos 2x - 2e^{-2x} \sin 2x$$
$$= 2e^{-2x} (\cos 2x - \sin 2x)$$

(x) **Find $\frac{d^2y}{dx^2}$ if $y^3 + 3ax^2 + x^3 = 0$.**

Ans

$$\frac{d}{dx}(y^3 + 3ax^2 + x^3) = \frac{d}{dx}(0)$$
$$3y^2\frac{dy}{dx} + 3(a\,(2x) + x^2\,(0)) + 3x^2 = 0$$
$$3y^2\frac{dy}{dx} + 3(2ax) + 3x^2 = 0$$
$$3y^2\frac{dy}{dx} + 6ax + 3x^2 = 0$$
$$3y^2\frac{dy}{dx} = -6ax - 3x^2$$
$$\frac{dy}{dx} = \frac{-3(2ax + x^2)}{3y^2}$$
$$\frac{dy}{dx} = \frac{-(2ax + x^2)}{y^2}$$

Differentiate Again.

$$\frac{d^2y}{dx^2} = -\left[\frac{y^2(2ax + x^2)' - (2ax + x^2)(y^2)'}{y^4}\right]$$
$$= \frac{-\left[y^2(2a + 2x) - (2ax + x^2)\,2y\frac{dy}{dx}\right]}{y^4}$$
$$= \frac{-\left[2(a + x)y^2 - (2ax + x^2).2y\left(\frac{-2ax + x^2}{y^2}\right)\right]}{y^4}$$
$$= \frac{-2(a + x)y^2 - \frac{2(2ax + x^2)(2ax + x^2)}{y}}{y^4}$$
$$= \frac{-2\,[(a + x)y^3 + (2ax + x^2)^2]}{y^4 . y}$$

Put $\quad y^3 = -3ax^2 - x^3$

$$= \frac{-2[(a + x)(-3ax^2 - x^3) + 4x^2a^2 + 4ax^3 + x^4]}{y^5}$$

$$= \frac{-2[(a+x)x^2(-3a-x) + x^2(4a^2+4ax+x^2)]}{y^5}$$

$$= \frac{-2x^2[-(a+x)(3a+x) + 4a^2+4ax+x^2]}{y^5}$$

$$= \frac{-2x^2[-(3a^2+4ax+x^2) + 4a^2+x^2+4ax}{y^5}$$

$$= \frac{-2x^2(a^2)}{y^5}$$

$$= \frac{-2a^2x^2}{y^5}$$

(xi) **Find y_2 if $y = \cos^3 x$.**

Ans

$$\frac{dy}{dx} = \frac{d}{dx}(\cos^3 x)$$

$$= 3\cos^2 x(-\sin x)$$

$$= -3\cos^2 x \sin x$$

$$= -3(1-\sin^2 x)\sin x$$

$$= -3\sin x + 3\sin^3 x$$

Differentiate again

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(-3\sin x + 3\sin^3 x)$$

$$= -3\cos x + 9\sin^2 x\cos x$$

$$= -3\cos x + 9(1-\cos^2 x)\cos x$$

$$= -3\cos x + 9\cos x - 9\cos^3 x$$

$$= 6\cos x - 9\cos^3 x$$

(xii) **Find $\frac{dy}{dx}$ if $y = ln\left(\frac{x^2-1}{x^2+1}\right)^{1/2}$.**

Ans Let

$$u = \frac{x^2-1}{x^2+1}$$

$$y = ln\sqrt{u}$$

$$= \frac{1}{2}\,ln\,u$$

$$\frac{dy}{du} = \frac{1}{2}\frac{1}{u}$$

$$= \frac{1}{2}\cdot\frac{1}{\frac{x^2-1}{x^2+1}}$$

$$\frac{dy}{du} = \frac{x^2+1}{2(x^2-1)}$$

As $\quad u = \dfrac{x^2-1}{x^2+1}$

Differentiate w.r.t 'x'

$$\frac{du}{dx} = \frac{d}{dx}\left(\frac{x^2-1}{x^2+1}\right)$$

$$= \frac{(x^2+1)(2x)-(x^2-1)(2x)}{(x^2+1)^2}$$

$$= \frac{2x(x^2+1-x^2+1)}{(x^2+1)^2}$$

$$= \frac{2x(2)}{(x^2+1)^2}$$

$$= \frac{4x}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \frac{x^2+1}{2(x^2-1)} \times \frac{4x}{(x^2+1)^2}$$

$$= \frac{2x(x^2+1)}{(x^2-1)(x^2+1)^2}$$

$$= \frac{2x}{(x^2-1)(x^2+1)}$$

$$= \frac{2x}{x^4-1}$$

3. Write short answers to any EIGHT (8) questions: 16

(i) Find δy and dy : $y = \sqrt{x}$, when x changes from 4 to 4.41.

Ans

$$y = \sqrt{x}$$

$$\frac{dy}{dx} = \frac{1}{2}(x)^{-1/2} \ . (1)$$

$$dy = \frac{1}{2\sqrt{x}}\,dx$$

As x changes from 4 to 4.41

So, $\quad x = 4$ and $\delta x = dx = 4.41 - 4 = 0.41$

$$dy = \frac{1}{2\sqrt{4}}(.41)$$

$$= \frac{1}{2\sqrt{(2)^2}}(.41)$$

$$= \frac{1}{4}(.41) = 0.1025$$

For change δx in x

$$y + \delta y = \sqrt{x + \delta x}$$

$$\delta y = \sqrt{x + \delta x} - y$$

$$= \sqrt{x + \delta x} - \sqrt{x}$$

If $x = 4 \;\&\; \delta x = .41$

$$\delta y = \sqrt{4 + .41} - \sqrt{4}$$

$$= \sqrt{4.41} - \sqrt{4}$$

$$= 2.1 - 2$$

$$= 0.1$$

(ii) Evaluate $\int \frac{e^{2x} + e^x}{e^x} dx$.

Ans

$$I = \int \left(\frac{e^{2x}}{e^x} + \frac{e^x}{e^x}\right) dx$$

$$= \int (e^x + 1)\, dx$$

$$= \int e^x\, dx + \int 1\, dx$$

$$I = e^x + x + c$$

(iii) Evaluate $\int (a - 2x)^{3/2} dx$.

Ans

$$\int (a - 2x)^{3/2} \times \left(\frac{-2}{-2}\right) dx$$

$$= \frac{-1}{2}\int (a - 2x)^{3/2}(-2)\, dx$$

$$= \frac{-1}{2}\left[\frac{(a - 2x)^{3/2+1}}{\frac{3}{2}+1}\right] + c$$

$$= \frac{-1}{2}\frac{(a - 2x)^{5/2}}{\frac{5}{2}} + c$$

$$= \frac{-1}{2}\cdot\frac{2}{5}(a - 2x)^{5/2} + c$$

$$= \frac{-1}{5}(a - 2x)^{5/2} + c$$

(iv) Evaluate $\int \frac{x+b}{(x^2+2bx+c)^{1/2}}\,dx.$

Ans
$$= \int \frac{x+b}{(x^2+2bx+c)^{1/2}}\,dx$$
$$= \int (x^2+2bx+c)^{-1/2}(x+b)\,dx$$
$$= \frac{1}{2}\int (x^2+2bx+c)^{-1/2}.(2x+2b)\,dx$$
$$= \frac{1}{2}\left[\frac{(x^2+2bx+c)^{-1/2+1}}{-\frac{1}{2}+1}\right]+c_1$$
$$= \frac{1}{2}\left[\frac{(x^2+2bx+c)^{1/2}}{\frac{1}{2}}\right]+c_1$$
$$= \frac{1}{\cancel{2}}.\cancel{2}(x^2+2bx+c)^{1/2}+c_1$$
$$= \sqrt{x^2+2bx+c}+c_1$$

(v) Evaluate $\int x\,e^x\,dx.$

Ans Integrate by parts
$$= x\,e^x - \int e^x(1)\,dx$$
$$= x\,e^x - e^x + c$$

(vi) Evaluate $\int e^x\left(\frac{1}{x}+\ln x\right)dx.$

Ans We can also write as
$$\int e^x\left(\ln x+\frac{1}{x}\right)dx$$

If $\quad f(x) = \ln x$

Then $\quad f'(x) = \frac{1}{x}$

So $\quad \int e^x(f(x)+f(x'))\,dx$
$$= e^x f(x) + c$$
$$= e^x \ln x + c$$

(vii) Evaluate $\int_{-1}^{3}(x^3+3x^2)\,dx.$

Ans $\int_{-1}^{3}(x^3+3x^2)\,dx$

$$= \int_{-1}^{3} x^3\,dx + 3\int_{-1}^{3} x^2\,dx$$

$$= \left|\frac{x^4}{4}\right|_{-1}^{3} + 3.\frac{1}{3}\left|x^3\right|_{-1}^{3}$$

$$= \frac{1}{4}\left[(3)^4 - (1)^4\right] + \left[3^3 - (-1)^3\right]$$

$$= \frac{1}{4}[81 - 1] + [27 + 1] = 20 + 28$$

$$= 48$$

(viii) **Evaluate $\int_{0}^{\pi/3} \cos^2\theta \sin\theta\, d\theta$.**

Ans

$$I = \int_{0}^{\pi/3} \cos^2\theta \sin\theta\, d\theta$$

If $f(\theta) = \cos\theta$

$f'(\theta) = -\sin\theta$

$$= -\int_{0}^{\pi/3} \cos^2\theta\,(-\sin\theta)\,d\theta$$

$$= -\left[\frac{\cos^3\theta}{3}\right]_0^{\pi/3} + c$$

$$= -\frac{1}{3}\left[\cos^3\frac{\pi}{3} - \cos^3 0\right] + c$$

$$= -\frac{1}{3}\left[\left(\frac{1}{2}\right)^3 - (1)^3\right] + c$$

$$= -\frac{1}{3}\left(\frac{1}{8} - 1\right) + c$$

$$= -\frac{1}{3}\left(\frac{7}{8}\right) \Rightarrow \frac{-7}{24}$$

(ix) **Find the area between the x-axis and the curve $y = 4x - x^2$ from $x = 0$ to $x = 4$.**

Ans Given equation of curve is $y = 4x - x^2$

To find x-intercept

Put $y = 0$

$$4x - x^2 = 0$$

$$x(4 - x) = 0$$

$$\boxed{x = 0, 4}$$

So given curve cuts x-axis at points (0, 0), (4, 0).
As $y = 4x - x^2 \geq 0$ for $0 \leq x \leq 4$
Thus the area bounded by given curve is above x-axis.
If A be the required area, then

$$A = \int_0^4 y\,dx$$

$$A = \int_0^4 (4x - x^2)\,dx$$

$$= \left[4\frac{x^2}{2} - \frac{x^3}{3}\right]_0^4$$

$$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$$

$$= \left[2(4)^2 - \frac{(4)^3}{3}\right] - \left[2(0)^2 - \frac{(0)^3}{3}\right]$$

$$= \left[2(16) - \frac{64}{3}\right] - [0 - 0]$$

$$= 32 - \frac{64}{3}$$

$$= \frac{96 - 64}{3}$$

$$\boxed{A = \frac{32}{3} \text{ square units}}$$

(x) Define differential equation.

Ans An equation having at least one derivative of a dependent variable w.r.t an independent variable.

(xi) Solve $\frac{dy}{dx} = \frac{y^2 + 1}{e^{-x}}$.

Ans By separating the variable, we have

$$\frac{dy}{y^2 + 1} = \frac{1}{e^{-x}}\,dx$$

$$\int \frac{1}{y^2 + 1}\,dy = \int e^x\,dx$$

$$\tan^{-1} y = e^x + c$$

$$y = \tan(e^x + c)$$

(xii) **Solve $\frac{dy}{dx} = 2x$.**

Ans By separating the variables, we have

$$dy = 2x\,dx$$

Integrate on both sides,

$$\int dy = \int 2x\,dx$$

$$y^2 = \frac{2x^2}{2} + c$$

$$y = x^2 + c$$

4. Write short answers to any NINE (9) questions: 18

(i) **Write down equation of straight line with x-intercept (2, 0) and y-intercept (0, –4).**

Ans Equation of a line whose none-zero x and y intercepts is:

$$\frac{x}{a} + \frac{y}{b} = 1$$

In x-intercept $a = 2, b = 0$
In y-intercept $a = 0, b = -4$
By putting values in the formula

$$\frac{x}{2} + \frac{y}{-4} = 1$$

$$\frac{2x - y}{4} = 1$$

$$2x - y = 4$$

$$2x - y - 4 = 0 \qquad \text{(Required equation)}$$

(ii) **Find an equation of a line bisecting 2^{nd} and 4^{th} quadrants.**

Ans The line passes through (0, 0) and having slope-1, so its equation is:

$$y = -x$$

(iii) **Find an equation of a line with x-intercept: –9 and slope: –4.**

Ans Given points are $(-9, 0)$ and $m = -4$
The equation of required line is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -4(x - (-9))$$

$$y = -4(x + 9)$$

$$y = -4x - 36$$

$$y + 4x + 36 = 0$$

(iv) Prove that if the lines are perpendicular, then product of their slopes = –1.

Ans As the lines are perpendicular to each other. So, Let,

$$\text{Inclimation of } l_1 = \alpha$$

$$\text{Inclimation of } l_2 = \alpha + 90$$

$$m_1 = \tan\alpha$$

$$m_2 = \tan(90 + \alpha) = -\cot\alpha$$

$$\text{Product of } m_1, m_2 = m_1 \times m_2$$

$$= \tan\alpha \times (-\cot\alpha)$$

$$= \tan\alpha \times \frac{-1}{\tan\alpha}$$

Product of their slopes = –1.

(v) Find the measure of angle between the lines represented by $x^2 - xy - 6y^2 = 0$.

Ans Here

$$a = 1, h = \frac{-1}{2}, b = -6$$

If θ is measure of the angle between the given lines, then

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

$$= \frac{2\sqrt{\left(\frac{-1}{2}\right)^2 - (1)(-6)}}{1 - 6} = \frac{2\sqrt{\frac{1}{4} + 6}}{-5} = \frac{2\sqrt{\frac{25}{4}}}{-5}$$

$$= -1$$

$$= \theta$$

$$= 135^\circ$$

$$\text{Acute angle between the lines} = 180^\circ - \theta$$

$$= 180^\circ - 135^\circ$$

$$= 45^\circ$$

(vi) Find focus and vertex of the parabola $y = 6x^2 - 1$.

Ans

$$y = 6x^2 - 1$$

$$\frac{y + 1}{6} = x^2$$

Let: $x = X$ and $Y = y + 1$

So $\frac{1}{6}Y = X^2$ (i)

Comparing equation (i) with $X^2 = 4aY$
we have,

$$4a = \frac{1}{6}$$

$$a = \frac{1}{24}$$

Vertex of equation (i) is (0, 0)

$$X = 0 \qquad Y = 0$$

$$x = 0 \qquad y + 1 = 0$$

$$y = -1$$

$$\boxed{\text{vertex} = (0, -1)}$$

$$\text{Focus of parabola} = (0, a)$$

$$= \left(0, \frac{1}{24}\right)$$

$$X = 0; \qquad Y = \frac{1}{24}$$

$$x = 0; \; y + 1 = \frac{1}{24}$$

$$y = \frac{1}{24} - 1$$

$$= \frac{1 - 24}{24}$$

$$y = \frac{-23}{24}$$

$$\text{Focus} = \left(0, \frac{-23}{24}\right)$$

(vii) Find equation of latus rectum of parabola $y^2 = -8(x - 3)$.

Ans $y^2 = -8(x - 3)$

Let $y = Y$ and $X = x - 3$

$y^2 = -8X$

Compare this equation with $Y^2 = 4aX$

We have

$$4a = -8$$

$$a = -2$$

Equation of Latus rectum $= x - a = 0$

$$x - 3 - (-2) = 0$$
$$x - 3 = -2$$
$$x = -2 + 3$$
$$\boxed{x = 1}$$

(viii) **Find an equation of an ellipse with foci $(\pm 3, 0)$ and minor axis of length 10.**

Ans Here foci $(\pm c, 0) = (\pm 3, 0)$

$$c = ae = 3$$
$$2b = 10$$
$$\Rightarrow \quad b = 5$$
As $\quad c^2 = a^2 - b^2$

Put values of b and c,

$$(3)^2 = a^2 - (5)^2$$
$$9 = a^2 - 25$$
$$9 + 25 = a^2$$
$$\Rightarrow \quad a^2 = 34$$
$$a = \sqrt{34}$$

Since major axis is along x-axis. So the equation of ellipse is $\frac{x^2}{34} + \frac{y^2}{25} = 1$.

(ix) **Find the foci and length of the latus rectum of the ellipse $9x^2 + y^2 = 18$.**

Ans
$$9x^2 + y^2 = 18$$
Divide by 18, we have

$$\frac{x^2}{2} + \frac{y^2}{18} = 1 \qquad \left[\because \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \; ; \; a > b\right]$$
$$a^2 = 18, b^2 = 2$$
$$c^2 = a^2 - b^2$$
$$= 18 - 2$$
$$c^2 = 16$$
$$c = \pm 4$$
$$\text{Foci} = (0, \pm c)$$
$$\boxed{F = (0, \pm 4)}$$

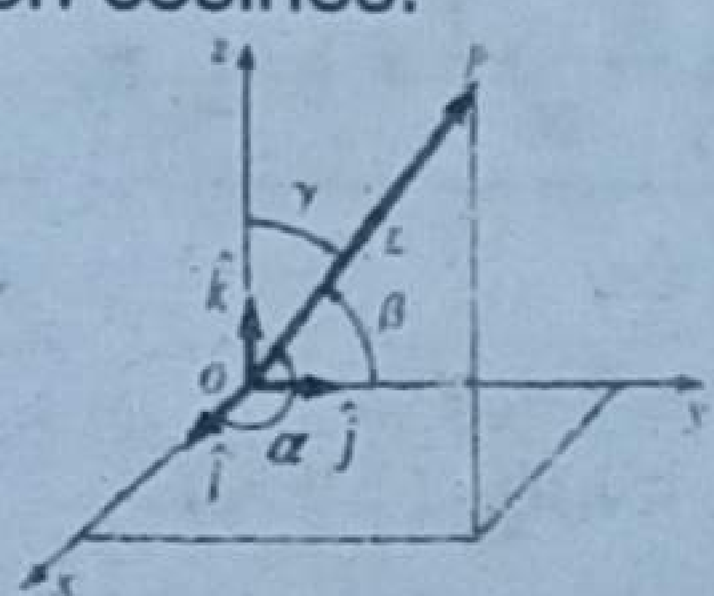
$$\text{Length of Latus Rectum} = \frac{2a^2}{b}$$
$$= \frac{2(18)}{\sqrt{2}}$$

$$= \frac{36}{\sqrt{2}}$$

(x) **Define direction angles and direction cosines of a vector.**

Ans Let $r = \overrightarrow{OP} = x\bar{i} + y\bar{j} + z\bar{k}$ be a non-zero vector. Let α, β, γ denoted the angles formed between r and the unit coordinate vectors.

α, β, γ are called direction angles and $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called direction cosines.



(xi) **Find the projection of vector $\underline{a}$ along vector $\underline{b}$ and projection of vector $\underline{b}$ along $\underline{a}$ when $\underline{a} = \hat{i} - \hat{k}$, $b = \hat{j} + \hat{k}$.**

Ans Projection of $\vec{a}$ along $\vec{b} = \frac{\underline{a}\,.\,\underline{b}}{|\underline{b}|}$

$$\underline{b} = j + k$$

$$|\underline{b}| = \sqrt{(1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$$

$$\underline{a}\,.\,\underline{b} = (\underline{i} - \underline{k})\,.\,(\underline{j} + \underline{k})$$
$$= (\underline{i} + 0\underline{j} - \underline{k})\,.\,(0\underline{i} + \underline{j} + \underline{k})$$
$$= 1 \times 0 + 0 \times 1 + (-1)(1)$$
$$= -1$$

Projection of $\vec{a}$ along $\vec{b} = \frac{-1}{\sqrt{2}}$.

(xii) **Find a vector perpendicular to each of the vectors $\underline{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\underline{b} = 4\hat{i} + 2\hat{j} - \hat{k}$.**

Ans Let $\underline{c}$ be the vector perpendicular to both $\underline{a}$ and $\underline{b}$.

$$\underline{c} = \underline{a} \times \underline{b}$$

$$\underline{c} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 1 & 1 \\ 4 & 2 & -1 \end{vmatrix}$$

$$= (-1-2)\,\underline{i} - (-2-4)\,\underline{j} + (4-4)\,\underline{k}$$
$$= -3\underline{i} + 6\underline{j} + 0\underline{k}$$

$\underline{c} = -3\underline{i} + 6\underline{j}$

(xiii) **Convert $2x - 4y + 11 = 0$ into slope intercept form.**

Ans

$$2x - 4y + 11 = 0$$
$$2x + 11 = 4y$$
$$\frac{x}{2} + \frac{11}{4} = y$$

SECTION-II

NOTE: Attempt any Three (3) questions.

Q.5.(a) Prove that $\lim_{x\to 0} \frac{a^x - 1}{x} = \log_e a$. (5)

Ans Put $a^x - 1 = y$

$$a^x = 1 + y$$
$$x = \log_a (1 + y)$$

When $x \to 0, y \to 0$

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \lim_{y\to 0} \frac{y}{\log_a(1 + y)}$$
$$= \lim_{y\to 0} \frac{1}{\frac{1}{y} \log_a (1 + y)}$$
$$= \lim_{y\to 0} \frac{1}{\log_a (1 + y)^{1/y}}$$
$$= \frac{1}{\log_a e}$$
$$= \log_e a$$

(b) Prove that $y\frac{dy}{dx} + x = 0$ if $x = \frac{1 - t^2}{1 + t^2}$, $y = \frac{2t}{1 + t^2}$. (5)

Ans

$$x = \frac{1 - t^2}{1 + t^2} \text{ and } y = \frac{2t}{1 + t^2}$$
$$x^2 + y^2 = \left(\frac{1 - t^2}{1 + t^2}\right)^2 + \left(\frac{2t}{1 + t^2}\right)^2$$
$$= \frac{(1 - t^2)^2 + (2t)^2}{(1 + t^2)^2}$$
$$= \frac{\cancel{1 + t^4} - 2t^2 + 4t^2}{\cancel{1 + t^4} + 2t^2}$$

$$= \frac{2t^2}{2t^2}$$

$$x^2 + y^2 = 1$$

Differentiating w.r.t 'x'

$$2x + 2y\frac{dy}{dx} = 0$$

$$2(x + y\frac{dy}{dx}) = 0$$

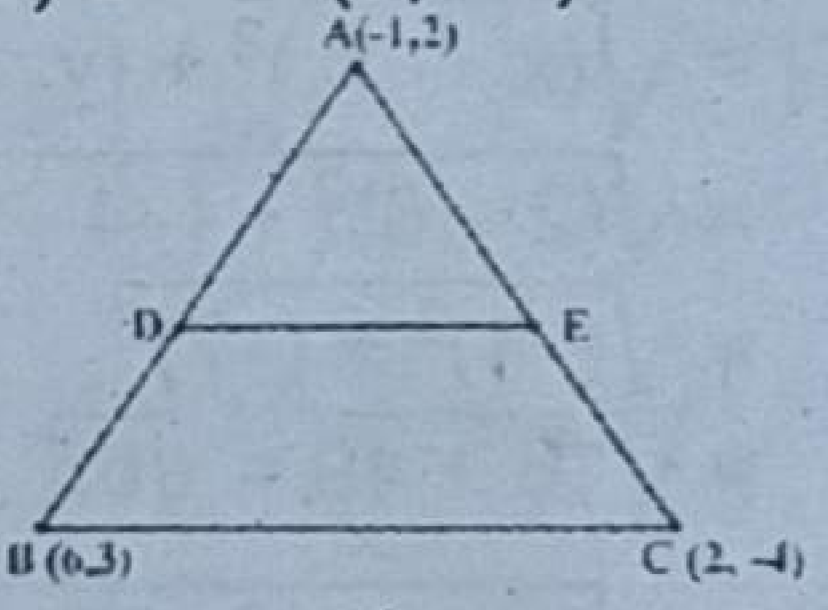
$$x + y\frac{dy}{dx} = 0$$ Hence proved.

Q.6.(a) Show that $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln(x + \sqrt{x^2 - a^2}) + c.$ (5)

Ans For Answer see Paper 2015 (Group-I), Q.6.(a).

(b) The points A(–1, 2), B(6, 3) and C(2, –4) are vertices of a triangle, then show that the line joining the mid-point "D" of $\overline{AB}$ and mid-point "E" of $\overline{AC}$ is parallel to $\overline{BC}$ and $\overline{DE} = \frac{1}{2}\overline{BC}$. (5)

Ans A(–1, 2), B(6, 3) and C(2, –4)



Then coordinates of $D = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$$= \left(\frac{-1 + 6}{2}; \frac{2 + 3}{2}\right)$$

$$D = \left(\frac{5}{2}, \frac{5}{2}\right)$$

Coordinates of $E = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$= \left(\frac{-1 + 2}{2}, \frac{2 - 4}{2}\right)$$

$$= \left(\frac{1}{2}, \frac{-2}{2}\right)$$

$$E = \left(\frac{1}{2}, -1\right)$$

$$\text{Slope of side } \overline{BC} = m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-4-3}{2-6} = \frac{-7}{-4} = \frac{7}{4}$$

$$\text{Slope of side } \overline{DE} = m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-1 - \frac{5}{2}}{\frac{1}{2} - \frac{5}{2}} = \frac{\frac{-2-5}{2}}{\frac{1-5}{2}} = \frac{\frac{-7}{2}}{\frac{-4}{2}}$$

$$= \frac{\frac{-7}{2}}{-2} = \frac{7}{4}$$

As $m_1 = m_2 = \frac{7}{4}$; So $\overline{DE} \parallel \overline{BC}$

$$|\overline{BC}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(2-6)^2 + (-4-3)^2}$$

$$= \sqrt{(-4)^2 + (-7)^2}$$

$$= \sqrt{16+49} = \sqrt{65}$$

$$|\overline{DE}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left(\frac{1}{2} - \frac{5}{2}\right)^2 + \left(-1 - \frac{5}{2}\right)^2}$$

$$= \sqrt{\left(\frac{-1-5}{2}\right)^2 + \left(\frac{-2-5}{2}\right)^2}$$

$$= \sqrt{\left(\frac{-4}{2}\right)^2 + \left(\frac{-7}{2}\right)^2}$$

$$= \sqrt{(-2)^2 + \left(\frac{-7}{2}\right)^2} = \sqrt{4 + \frac{49}{4}}$$

$$= \sqrt{\frac{16+49}{4}} = \sqrt{\frac{65}{4}} = \sqrt{\frac{65}{2}}$$

$$|\overline{DE}| = \frac{1}{2}\sqrt{65} = \frac{1}{2}|\overline{BC}| \quad \text{Hence proved.}$$

Q.7.(a) Evaluate $\int_0^{\pi/4} \cos^4 t \, dt$. (5)

Ans

$$= \frac{1}{4}\int_0^{\pi/4} 4\cos^4 t \, dt$$

$$= \frac{1}{4}\int_0^{\pi/4} (2\cos^2 t)^2 \, dt$$

$$= \frac{1}{4}\int_0^{\pi/4} (1+\cos 2t)^2 \, dt$$

$$= \frac{1}{4}\int_0^{\pi/4} (1+\cos^2 2t + 2\cos 2t) \, dt$$

$$= \frac{1}{4}\int_0^{\pi/4} \left(1+\frac{1+\cos 4t}{2} + 2\cos 2t\right) dt$$

$$= \frac{1}{4}\int_0^{\pi/4} \left(1+\frac{1}{2}+\frac{1}{2}\cos 4t + 2\cos 2t\right) dt$$

$$= \frac{1}{4}\int_0^{\pi/4} \left(\frac{3}{2}+\frac{1}{2}\cos 4t + 2\cos 2t\right) dt$$

$$= \frac{1}{4}\left[\frac{3}{2}\int_0^{\pi/4} 1\,dt + \frac{1}{2}\int_0^{\pi/4} \cos 4t\,dt + 2\int_0^{\pi/4} \cos 2t\,dt\right]$$

$$= \frac{1}{4}\left[\frac{3}{2}|t|_0^{\pi/4} + \frac{1}{2}\left|\frac{\sin 4t}{4}\right|_0^{\pi/4} + \cancel{2}\left|\frac{\sin 2t}{\cancel{2}}\right|_0^{\pi/4}\right.$$

$$= \frac{1}{4}\left[\frac{3}{2}\left[\frac{\pi}{4}-0\right] + \frac{1}{8}\left[\sin 4\frac{\pi}{4} - \sin 0\right] + \left(\sin 2\frac{\pi}{4} - \sin 0\right)\right]$$

$$= \frac{1}{4}\left[\frac{3\pi}{8} + \frac{1}{8}(\sin\pi - 0) + \left(\sin\frac{\pi}{2} - 0\right)\right]$$

$$= \frac{1}{4}\left[\frac{3\pi}{8} + \frac{1}{8}(0-0) + (1-0)\right]$$

$$= \frac{1}{4}\left[\frac{3\pi}{8} + 1\right] = \frac{1}{4}\left(\frac{3\pi+8}{8}\right)$$

$$= \frac{3\pi+8}{32}$$

(b) **Graph the feasible region of system of linear inequalities and find the corner points** (5)

$2x + 3y \leq 18,\ x + 4y \leq 12,\ 3x + y \leq 12 \qquad x \geq 0, y \geq 0$

Ans

$$2x + 3y \leq 18 \quad \text{(i)}$$
$$x + 4y \leq 12 \quad \text{(ii)}$$
$$3x + y \leq 12 \quad \text{(iii)}$$

In equation (iii), put $y = 0$, $x = 0$

$$3x + 0 \leq 12 \qquad 3(0) + y \leq 12$$
$$3x \leq 12 \qquad y \leq 12$$
$$x \leq \frac{12}{3} = 4$$

Corner point = (4, 0) (0, 12)

Put $x = 0$, $y = 0$ in equation (ii),

$$0 + 4y \leq 12 \qquad x + 4(0) \leq 12$$
$$4y \leq 12 \qquad x \leq 12$$
$$y \leq \frac{12}{4}$$
$$y \leq 3$$

Corner points (0, 3) (12, 0)

Compare equations (ii) and (iii),

Multiply eq. (ii) by '3' and then subtract

$$\begin{array}{r} 3x + 12y \leq 36 \\ 3x \pm \quad y \leq 12 \\ \hline 11y \leq 24 \end{array}$$
$$y \leq \frac{24}{11}$$

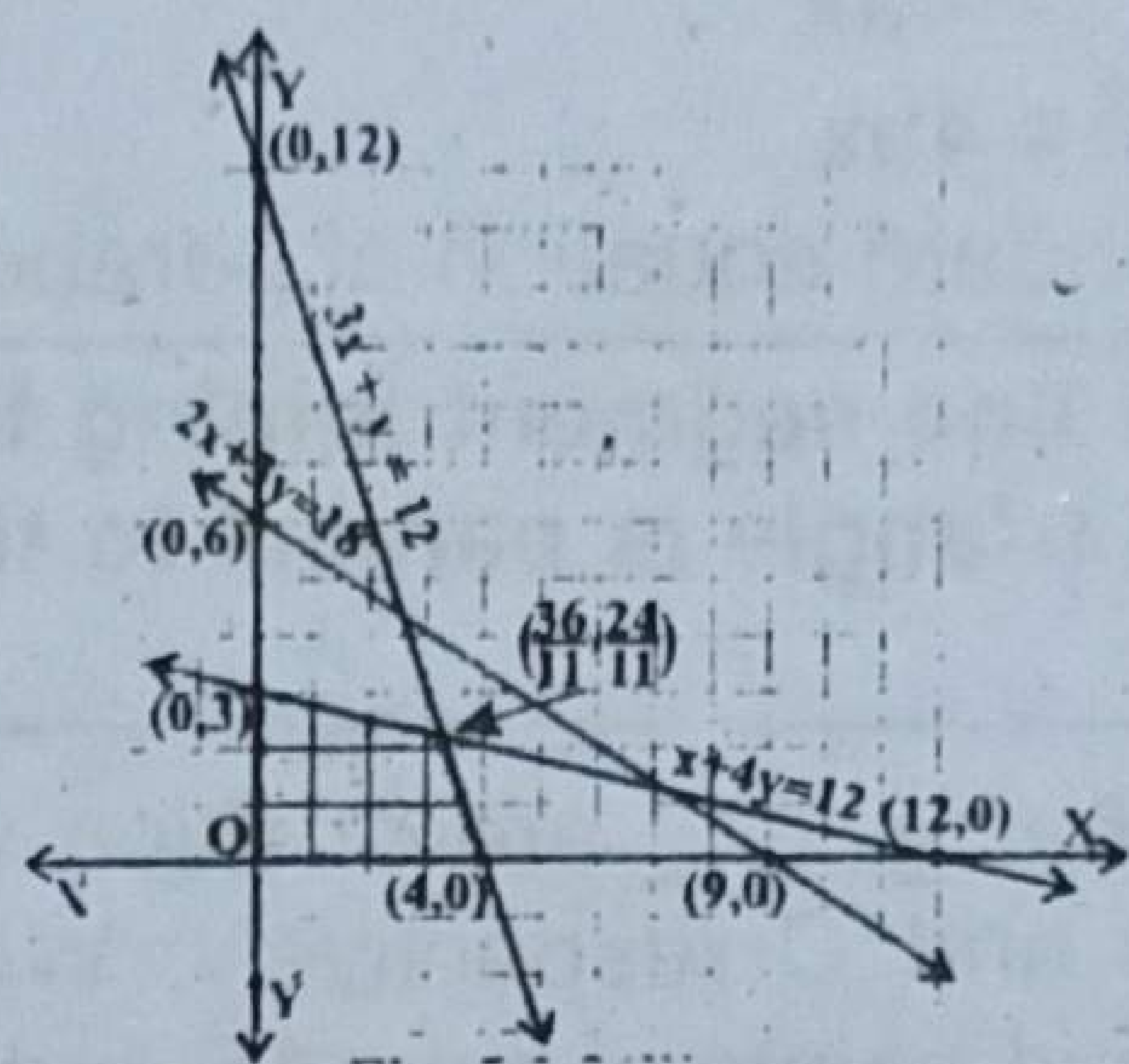
Put $y \leq \frac{24}{11}$ in eq. (iii),

$$3x + \frac{24}{11} \leq 12$$
$$3x \leq 12 - \frac{24}{11}$$
$$3x \leq \frac{132 - 24}{11}$$
$$x \leq \frac{108}{3 \times 11}$$
$$x \leq \frac{108}{33}$$

$$x \le \frac{36}{11}$$

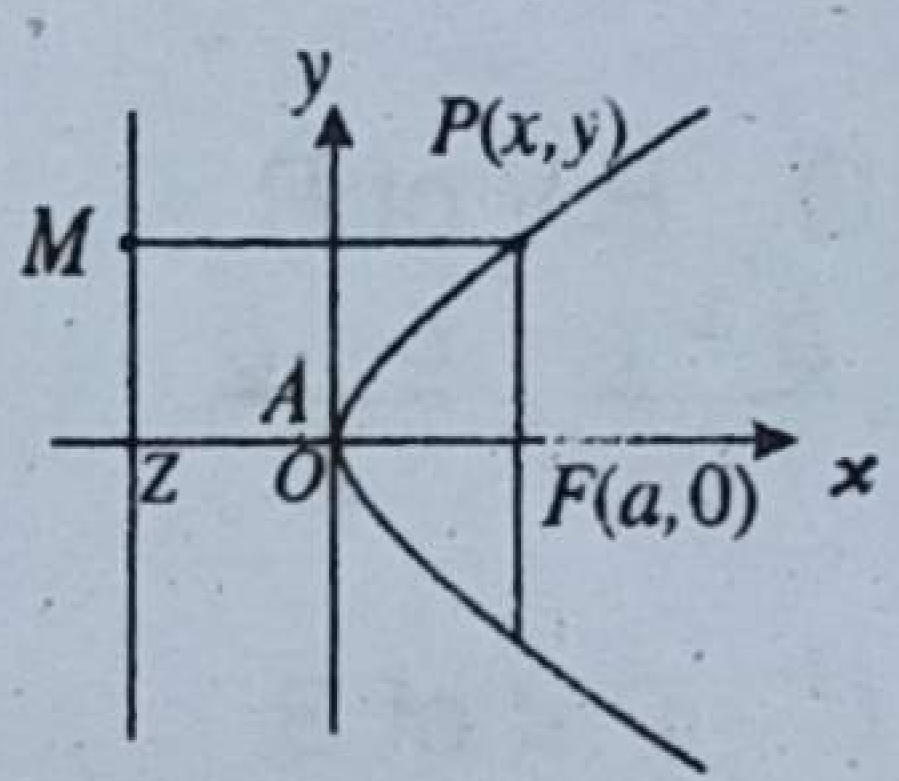
Corner points $\left(\frac{24}{11}, \frac{36}{11}\right)$

So corner points are $(0, 0)$ $(4, 0)$ $(0, 3)$ $\left(\frac{24}{11}, \frac{36}{11}\right)$.



Q.8.(a) Find an equation of parabola having its focus at the origin and directrix parallel to y-axis. (5)

Ans Let focus $f(a, \theta)$ be the focus of parabola and $x = -a$ the equation of directrix.



Also let $P(x, y)$ be a point on the parabola and M be the point on directrix. Then

$$\frac{|\overline{PF}|}{|\overline{PM}|} = 1$$

$$|\overline{PF}| = |\overline{PM}|$$

$$M = x = -a$$

$$|PM| = x + a$$

$$|PF| = \sqrt{(x-a)^2 + (y-0)^2}$$

$$= \sqrt{(x-a)^2 + y^2}$$

As $|PF| = |PM|$

$$\sqrt{(x-a)^2 + y^2} = x + a$$

Taking square on both sides,

$$\left(\sqrt{(x-a)^2+y^2}\right)^2 = (x+a)^2$$
$$(x-a)^2+y^2 = (x+a)^2$$
$$y^2 = (x+a)^2-(x-a)^2$$
$$= (x^2+a^2+2ax)-(x^2+a^2-2ax)$$
$$= x^2+a^2+2ax-x^2-a^2+2ax$$
$$= 4ax$$
$$y^2 = 4ax$$

which is the standard equation of parabola.

(b) Prove that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and half as long. (5)

Ans We suppose that $\underline{a}$, $\underline{b}$, $\underline{c}$ are position vectors of triangle having vertices A, B and C respectively. We further suppose that P and Q be the mid-points of side AB and AC.

As P.V of $P = \dfrac{\underline{a}+\underline{b}}{2}$

and P.V of $Q = \dfrac{\underline{a}+\underline{c}}{2}$

$$\overrightarrow{PQ} = \text{P.V of } Q - \text{P.V of } P$$
$$= \frac{\underline{a}+\underline{c}}{2} - \frac{\underline{a}+\underline{b}}{2} = \frac{\underline{c}-\underline{b}}{2} \qquad \text{(i)}$$

Now $\overrightarrow{BC} = \text{P.V of } C - \text{P.V of } B$

$$= c - b \qquad \text{(ii)}$$

$$\frac{\overrightarrow{BC}}{2} = \frac{\underline{c}-\underline{b}}{2}$$

Therefore, from (i) and (ii),

$$\overrightarrow{PQ} = \frac{\overrightarrow{BC}}{2} = \frac{1}{2}\overrightarrow{BC} \qquad \text{(iii)}$$

Therefore, vector $\overrightarrow{PQ} \parallel \overrightarrow{BC}$ and it is clear from (iii) length of $\overrightarrow{PQ}$ is half as long as $\overrightarrow{BC}$.

Q.9.(a) Find the centre, foci, eccentricity, vertices and equations of directrices of $\dfrac{y^2}{4} - x^2 = 1$. (5)

Ans For Answer see Paper 2018 (Group-I), Q.9.(a).

(b) **Find the value of α, in the coplanar vectors $\alpha\hat{i} + \hat{j}$, $\hat{i} + \hat{j} + 3\hat{k}$, $2\hat{i} + \hat{j} - 2\hat{k}$.** (5)

Ans Let $\underline{u} = \alpha\underline{i} + \underline{j}$

$\underline{v} = \underline{i} + \underline{j} + 3\underline{k}.$

$\underline{w} = 2\underline{i} + \underline{j} - 2\underline{k}.$

Given $\underline{u} \cdot (\underline{v} \times \underline{w}) = 0$

$$\begin{vmatrix} \alpha & 1 & 0 \\ 1 & 1 & 3 \\ 2 & 1 & -2 \end{vmatrix} = 0$$

$\alpha(-2 - 3) - 1(-2 - 6) + 0\,(1 - 2) = 0$

$-5\alpha + 8 = 0$

$-5\alpha = -8$

$$\alpha = \frac{-8}{-5}$$

$$\boxed{\alpha = \frac{8}{5}}$$